Pictorial Illustration of Fourier Transform as Applied to Digital Fault Recorder

By
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Abstract:

The concept of Fourier Transform (FT) is buried under an abstract mathematical formula. As it applies to Digital Fault Recorder (DFR), its limitation as well as its usefulness is often misunderstood. This paper attempts to bring the concept to the forefront by means of pictorial illustrations of how FT is applied to sinusoidal data. Its practical application in DFR such as phasor measurement, harmonics, and inter-harmonics will be illustrated.

Introduction:

This paper attempts to make the abstract concept of FT more understandable. It have been the authors experience that many engineering students question what they actually learn after taking a class of FT. They know, after the class, that they can transform data from time to frequency and back using FT, and it is useful to work in the frequency domain since it can solve some problem easier than the time domain. But how does FT really works, and is it comprehensible?

Vectors versus sinusoidal data:

Before I jump into FT, I would like to talk about sinusoidal data which is what DFR records (see Figure-1). We know recorded power signals are sinusoidal. Sinusoidal data goes up and down. It is related to a rotating vector.

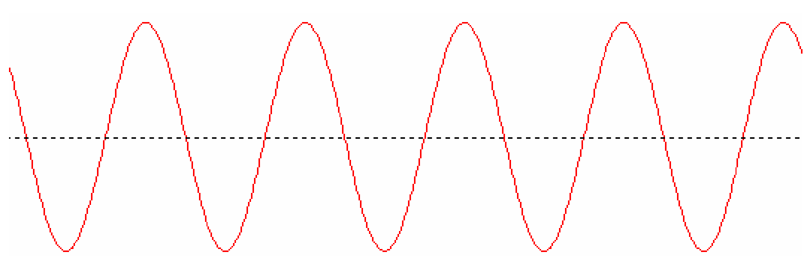


Figure-1: Recorded power signal is sinusoidal

Figure-2 shows the relationship between vector and sinusoid. On the left, vector **A** is rotating counter-clockwise starting at t=0 at the x-axis. On the right, it shows what the amplitude of **A** is projected onto the y-axis and the x-axis. You can imagine that the rotating vector **A** is the motor and the sinusoidal signal at X and Y is electrical power going through the electrical wires. As you can see from Figure-2, at t=0, X is at its maximum and Y is zero. When **A** reach 90 degrees, Y is at its maximum and X is at zero.

And it goes on and on. And we call the signal at X cosine and at Y sine. That means we need two sinusoidal data at any instant in time to truly represent a rotating vector.

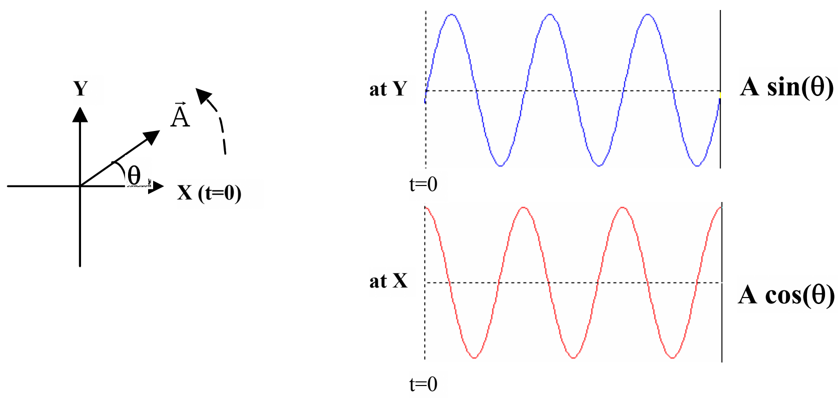


Figure-2: Vector vs. sinusoid

But recorded power signals have only one sinusoid. How can we use only one sinusoidal data to represent a rotating vector? We can if we assume the vector did not change its amplitude and speed in the time period of interest. Then we can shift back 90 degrees to get the sine (see Figure-3). Using this assumption, we can determine the phase of the rotating vector (or signal). This works, but is not accurate. Power signals have offsets and harmonics to alter the true amplitude of the signal at any instant of time. And this leads us to use FT for power signal analysis.

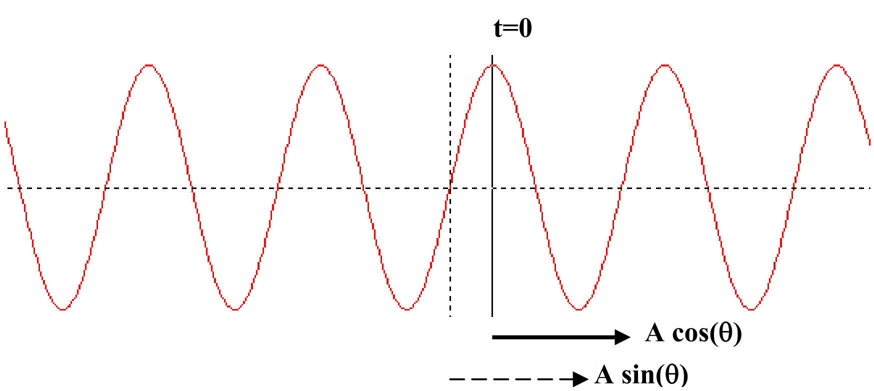


Figure-3: Hidden vector in sinusoid

Applying FT to sinusoidal data:

The basic assumption of FT:

- The signal is periodical.
- The signal can be expressed by the sum of sinusoids with different amplitudes and frequencies.

Therefore, the requirement of a steady signal for the period of interest as we talked about in Figure-3 also applies to FT. The second assumption simply means FT is perfect for harmonic analysis.

The basic formula of FT (see Equation-1) does not look like much but it encompasses a very abstract concept. It is the objective of this paper to illustrate the concept behind this formula in a more understandable manner.

Equation-1: $\mathbf{F(\omega) = \int f(t)\, e^{-j\omega t}\, dt}$

For the power signal which is pre-dominated by sinusoids, we can re-write Equation-1 to a less abstract one (Equation-2). We are not going into detail how it is derived. For digital fault recorder, the data recorded is digitized and therefore discrete. We can rewrite Equation-2 to be discrete as shown in Equation-3 by simply changing the integral symbol with the summation symbol and dropping the dt. Since the continuous and discrete formulas are interchangeable (in all practical matters), for convenience, we will use mostly the continuous one in this paper. Also notice the power signal function f(t) is the sum of signals with different frequencies.

Continuous:

Equation-2: $\mathbf{F(\omega_k) = 2 \int f(t) \cos(\omega_k t)\, dt - 2\, j \int f(t) \sin(\omega_k t)\, dt}$

Discrete:

Equation-3: $\mathbf{F(\omega_k) = 2 \sum f(t) \cos(\omega_k t) - 2\, j \sum f(t) \sin(\omega_k t)}$

where $f(t)=\sum f(\omega_i t)$ is recorded power signal.

The FT can be made easier to understand if we can replace it with vectors.

Let $\mathbf{u} = u(\omega_k t) = \cos(\omega_k t) - j \sin(\omega_k t)$ and $\mathbf{A}_i = f(\omega_i t)$, we can rewrite Equation-2 as Equation-4 and Equation-5.

Equation-4: $\mathbf{F\,(\omega_k) = 2 \int \sum f(\omega_i t)\, u(\omega_k t)\, dt = \sum F_i\,(\omega_k)}$

where

Equation-5: $\mathbf{F_i\,(\omega_k) = 2 \int f(\omega_i t)\, u(\omega_k t)\, dt = 2 \int A_i\, u\, dt}$

u is a unity vector with an amplitude equal to one. $\mathbf{A_i}$ is not a vector but if we simply relate it to a vector, we will see later in this paper that it will make FT easier to remember and understand.

Let $f(\omega_i t) = A \cos(\omega_i t+\varphi)$ and $F_i(\omega_k) = Fc + Fs$ and substitute them into Equation-5. We get Fc in Equation-6 and Fs in Equation-7. They are the cosine and sine parts of FT.

Equation-6: $\mathbf{Fc = 2 \int A \cos(\omega_i t+\varphi) \cos(\omega_k t)dt}$

Equation-7: $\mathbf{Fs = -2j \int A \cos(\omega_i t+\varphi) \sin(\omega_k t)dt}$

$F_i(\omega_k=\omega_i=\omega)$:

Now we want to see what happen if $\omega_k=\omega_i=\omega$.

Let $\theta=\omega t$, we get the following Fc and Fs.

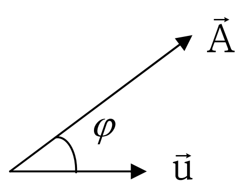
Equation-8: $\mathbf{Fc = 2 \int_{2\pi} A \cos(\theta+\varphi) \cos(\theta) d\theta = 2\pi A \cos(\varphi)}$

Equation-9: $\mathbf{Fs = -2j \int_{2\pi} A \cos(\theta+\varphi) \sin(\theta) d\theta = -j\, 2\pi A \sin(\varphi)}$

Integrate Fc and Fs over the period of 2π. We get the form (Equation-10) many of us will be familiar with. It is just a vector with angle φ multiplied by 2π.

Equation-10: $\mathbf{F_i(\omega) = 2\pi A[\cos(\varphi) + j \sin(\varphi)] = 2\pi\, A\angle\varphi}$

If we let n = number of samples = 2π, then $F_i(\omega) = n\,\mathbf{A}$ where **A** is the vector. If we try to picture it, we can see that **A** is related to **u** with a fixed angle φ (see Figure-4). This is how a phase angle of a power signal is being calculated in DFR.



where **u** is the reference frame

Figure-4: $F_i(\omega_k= \omega_p = \omega)$ shows as vectors

$F_i(\omega_k \neq \omega_i)$:

But what happen if $\omega_k \neq \omega_i$?

Let $\theta = \omega_0 t$, $\omega_k t = k\theta$ and $\omega_i t = m\theta$, where m is integer. And for simplicity, let $\varphi=0$. Then we get the cosine and sine of FT as show in Equation-11 and Equation-12.

Equation-11: $$Fc = 2 \int_{2\pi} A \cos(m\theta) \cos(k\theta)\, d\theta$$

Equation-12: $$Fs = -j2 \int_{2\pi} A \cos(m\theta) \sin(k\theta)\, d\theta$$

We know
$\cos(A)\cos(B) = \frac{1}{2} (\cos(A+B) + \cos(A-B))$,
$\sin(A)\sin(B) = \frac{1}{2} (\sin(A+B) + \sin(A-B))$, and
let a=m+k and b=m-k. We get

Equation-13: $$Fc = A \int_{2\pi} \cos(a\theta) + \cos(b\theta)\, d\theta = 0$$

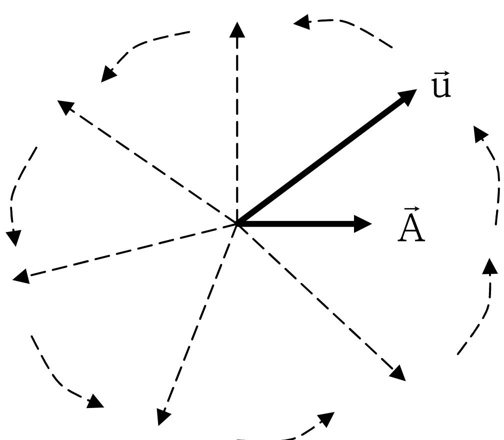
Equation-14: $$Fs = -jA \int_{2\pi} \sin(a\theta) - \sin(b\theta)\, d\theta = 0$$

Then we integrate Fc and Fs over the period of 2π. We get Fc=0 and Fs=0.

Therefore,

Equation-15: $$F_i(\omega_k \neq \omega_p) = 0$$

If we try to picture what the vectors look like if the frequencies are different, we will find that it looks like the vectors show in Figure-5. Let the vector **A** fixed. We will see the vector **u** is simply rotating around vector **A**. Integrating the rotation cancels out the effect. Therefore, we get F_i equals zero.



u is rotating

A is fixed

Figure-5: $F_i(\omega_k \neq \omega_i)=0$

Therefore, we come to the following conclusion.

We know

Equation-16: $$F(\omega_k) = F_i(\omega_k = \omega_i) + \sum F_j(\omega_k \neq \omega_j)$$

Since $F_i(\omega_k = \omega_i) = 2\pi A\angle\varphi$ and $F_j(\omega_k \neq \omega_j) = 0$, we got

Equation-17: $$F(\omega_k) = F_i(\omega_k = \omega_i) = n A\angle\varphi$$

Therefore, we can conclude that Fourier transforming a signal with frequency k is the same as extracting the sub-signal with frequency k from the signal. In other words, Fourier Transforming a signal with frequency k is the k-harmonic.

Harmonics (f_0=60Hz):

If we assume a fundamental frequency of 60Hz, then the Fourier Transform of 60Hz is simply equal to n times vector $\mathbf{A_1}$ which is the fundamental vector and doing that on 120Hz is the 2nd Harmonic vector, and 180Hz is the 3rd Harmonics vector and so on.

F(60Hz) = fundamental = $nA_1\angle\varphi_1$
F(120Hz) = 2nd Harmonic = $nA_2\angle\varphi_2$
F(180Hz) = 3rd Harmonic = $nA_3\angle\varphi_3$
…

Inter-harmonics (f_0=60Hz):

We talked about Harmonics above. Now you might ask what about inter-harmonics? If we assume 60Hz is the fundamental frequency, then an example of inter-harmonic is 61Hz, since it is not the integer multiple of 60. If we want to compare F(60Hz) with F(61Hz), we will need a resolution of 1Hz. In other words, the period of the integration needs to be 1 second. This might be a surprise to some that to find inter-harmonics, we need a longer period instead of a higher sampling rate. In theory, the real fundamental frequency in this case is 1Hz instead of 60Hz.

F(61Hz) is inter-harmonics

T needs to be at least 1/1Hz = 1 second,

since the resolution needed is 1Hz

FT Strengths

- Takes off offset naturally
- Cancels out other frequency naturally

FT Limitations

- Only for periodic signals
- Any frequency not the integer multiple of the lowest frequency defined by the window cannot be represented accurately

Practical Examples:

Now I will show some examples related to digital fault recorder. In general, in discrete Fourier Transforming a power signal, we are looking for harmonic contents of the signal as show in the bar graph in Figure-6. The bottom graph with a single spike is a Fast Fourier Transform (FFT) curve. At first sight, the FFT result is really showing us nothing.

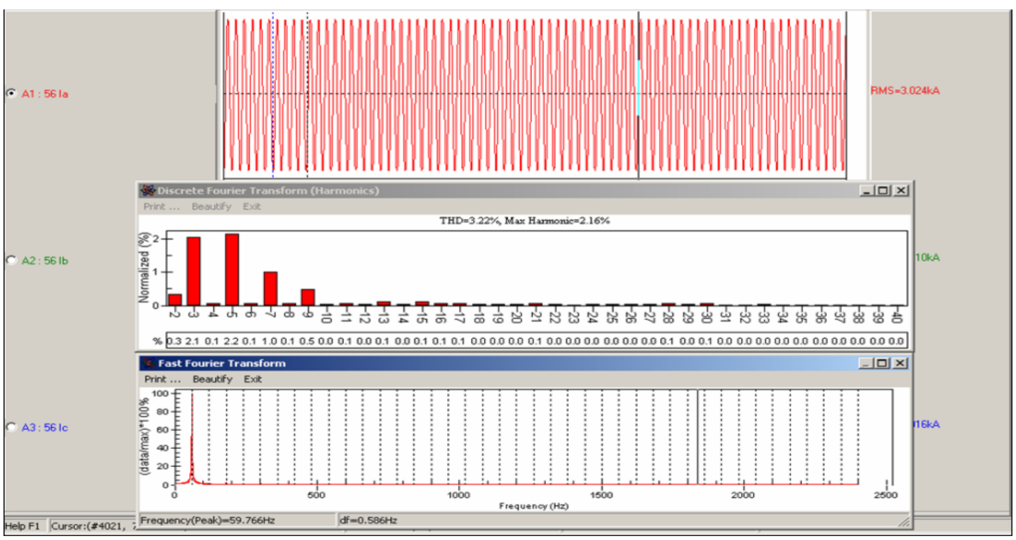


Figure-6: Harmonic versus FFT

But zooming into the FFT (see Figure-7), we see something resembled to the harmonic bar graph. There are spikes of 3rd, 5th, 7th and 9th harmonic. Although there is no inter-harmonic in this signal, we can clearly see that FFT will show inter-harmonics if it happens to be in the signal. We are not going into detail of FFT, but we simply point out that FFT is equivalent to DFT but can be processed much faster. But DFT is much simpler and targets particular frequency more directly and uses much less memory which is very crucial when applying to digital signal processor (DSP) or micro-controller.

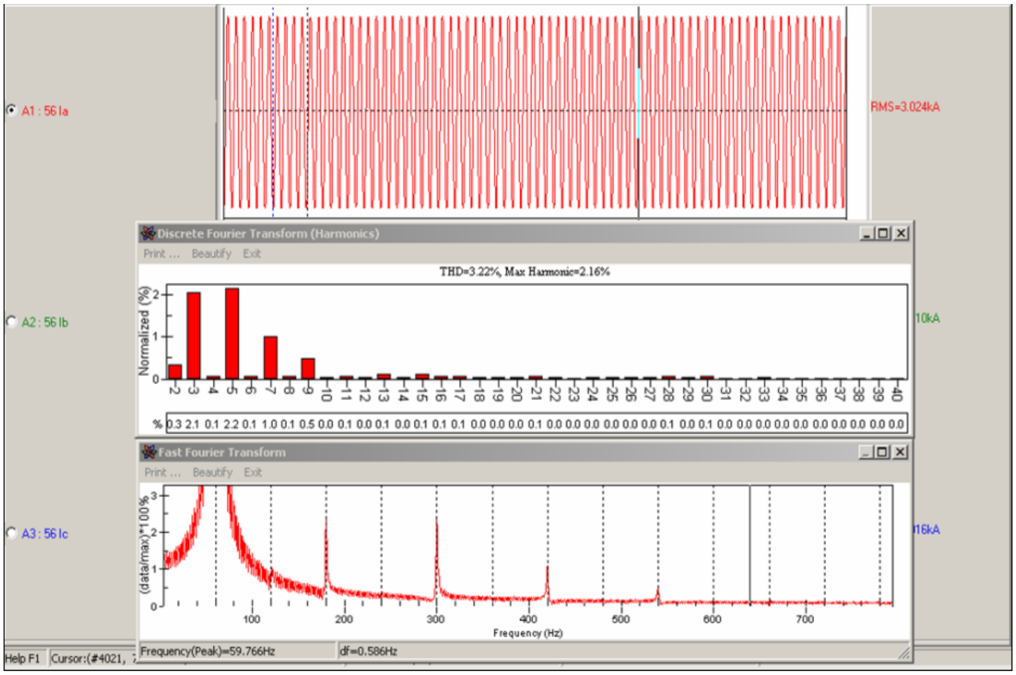


Figure-7: Zoom-In FFT

Another example of the application of Fourier Transform is the phasor measurement. The phasors are calculated using FT in the period between the solid line and the dot-dot line in Figure-8.

Figure-9 shows the same data but the period of calculation is right inside the fault. We can see current vector diagram indicating a large current flow and about 45 degrees shift in the phase.

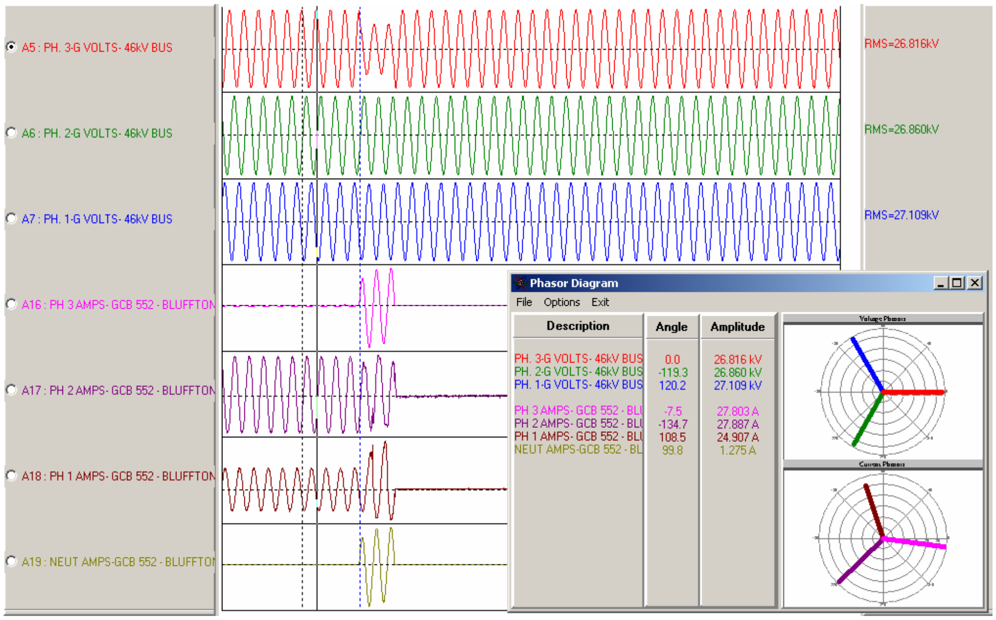


Figure-8: Phasor diagram versus sinusoidal data

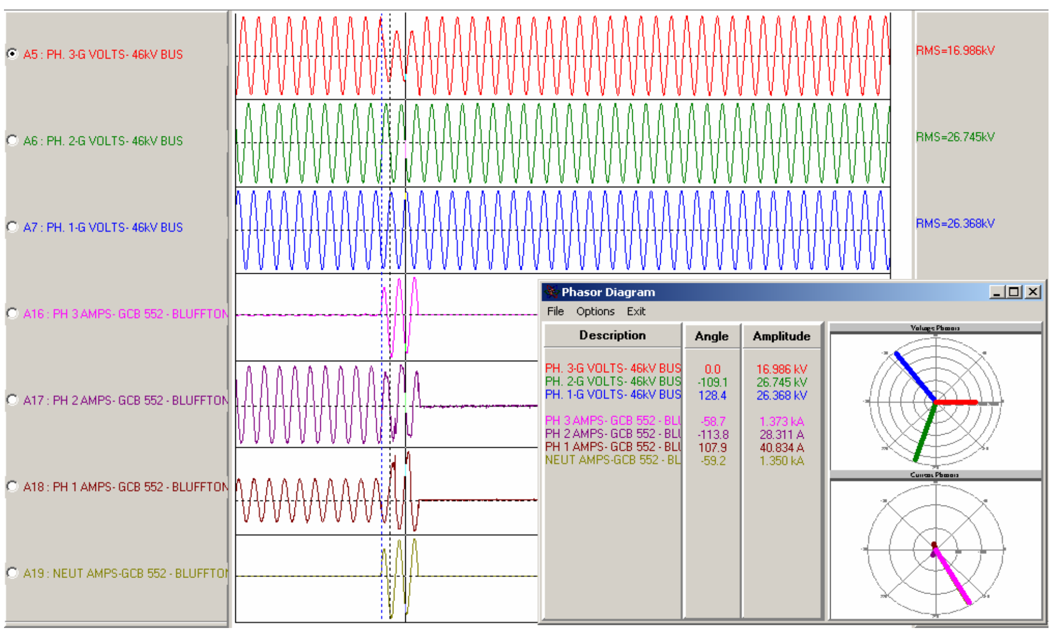


Figure-9: Phasor during a fault

Conclusions:

- Fourier Transform can be presented in vector format for easier comprehension
- Fourier transforming a signal with the same frequency as the transformation produces a vector
- Fourier transforming a signal with a different frequency as the transformation produces zero
- The resolution of the frequency in Fourier Transform is the fundamental frequency

Biography:

Dr. Wing-Kin Wai is the Vice President and co-founder of APP Engineering Inc. manufacture of digital fault recorder. He has been in the fault recorder industry for over 8 years. He is one of the two main designers of APP fault recorders. He received his BSEE in 1988 from Ohio University, Athens, Ohio, and MSEE (study focused on Artificial Intelligence) in 1990 and Ph.D. in 1993 both from Purdue University, West Lafayette, Indiana. He was born in Hong Kong. In 1982, he went to Canada to study grade 13 and in 1983 took one year in EET at the Northern College of Applied Arts and Technology, Kirkland Lake, Ontario before he came to America in 1984. He received his permanent residency in 1998 through the process called National Interest Wavier recommend by U.S. Department of Energy and Professors from Purdue University for his contribution in ripeness sensing of fruit using the Nuclear Magnetic Resonance theory and technology during his Ph.D. study. He became a U.S. citizen in 2004 and co-founded APP Engineering Inc. in 2005.

Greg Bradley is the President and co-founder of APP Engineering Inc. manufacture of digital fault recorders. He has been in the fault recorder industry for over 8 years. He is one of the two main designers of APP fault recorders. He received his BSEET in 1984 from Purdue University, Indianapolis, Indiana. He has over 23 years of sales, management, and original equipment manufacturing experience. He served on the Purdue University Indianapolis, EET industrial advisory committee and has been a TRUC committee member for the last 4 years.