

# Analysis on the Effect of Sub-Harmonic Frequencies on P Class Phasor Measurements

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## Abstract

The introduction of power electronic devices, series capacitors, wind and PV installations into the power grid has become common in recent times. A consequence to the use of these technologies are anomalies in the sub-harmonic frequency spectrum. As PMUs can be strategically placed at different parts of the power grid which are spread over a wide geographical area, measurements obtained from them have proven to be useful for both real-time and offline analysis. The IEEE C37.118.1a-2011 [1] standard and its recent 2018 update, the IEC/IEEE 60255-118-1-2018 [2], are widely accepted as benchmark sources for the performance of a PMU and provide reference models for both P and M class variants. While the standard specifies performance requirements under steady and dynamic conditions, behaviour under practical scenarios can be vastly different due to several factors, one of which being the presence of sub-harmonic frequencies. This paper presents a study on the effects that such sub-harmonic frequencies may have on P class phasor measurements captured during various power system conditions. A power system model is designed and subjected to several system conditions involving the presence of sub-harmonic frequencies. The three-phase voltage and current signals obtained from the model are used to generate P class phasor measurements as per the IEEE reference model and are further employed for analytical purposes specific to each scenario. A critical analysis is presented between data obtained via this method as well as dedicated fault recording and analysis equipment. The study highlights the expected challenges associated with the usage of these measurements for analysis applications and the considerations that need to be made.

## 1 Introduction – Characteristics of P class phasor measurements

Phasor measurement involves the estimation of two fundamental quantities of a signal, the magnitude and the phase angle. The IEEE/IEC 60255-118-1 standard specifies two classes of PMUs – protection class (P) and measurement class (M), along with reference measurement algorithms [2]. A key factor involved in the design of a PMU compliant to this standard is that measurements must be made available at defined time intervals in a consistent manner. This implies that constraints are imposed on the selection of the portion of the signal, otherwise known as the window, & the filtering mechanism.

Table 1 lists some of the key characteristics of the P class PMU & their significance.

**Table 1:** Properties of P class PMU

| Characteristic     | Significance  |
|--------------------|---|
| Window length      | 2 nominal cycles - Short acquisition window implies minimal filtering.  |
| Frequency tracking | Non frequency tracking & fixed to 50/60 Hz nominal – Accuracy compromised with off-nominal frequency signals. |
| Filter             | Low pass filter with fixed conversion frequency – no protection against signals < nominal frequency.          |

|                        |  |
|------------------------|--|
| Response to modulation | Can faithfully reproduce modulations up to a limited frequency range. Unsuitable for sub-synchronous resonance analysis. |
|------------------------|--|

The fast response offered by the P class PMU is offset by the characteristics mentioned above which gain specific importance in the presence of sub-harmonic frequencies as elaborated in the subsequent sections.

**2 Sub-harmonic frequencies in the power system**

Sub-harmonic frequencies in the power system can be broadly classified into 2 categories -

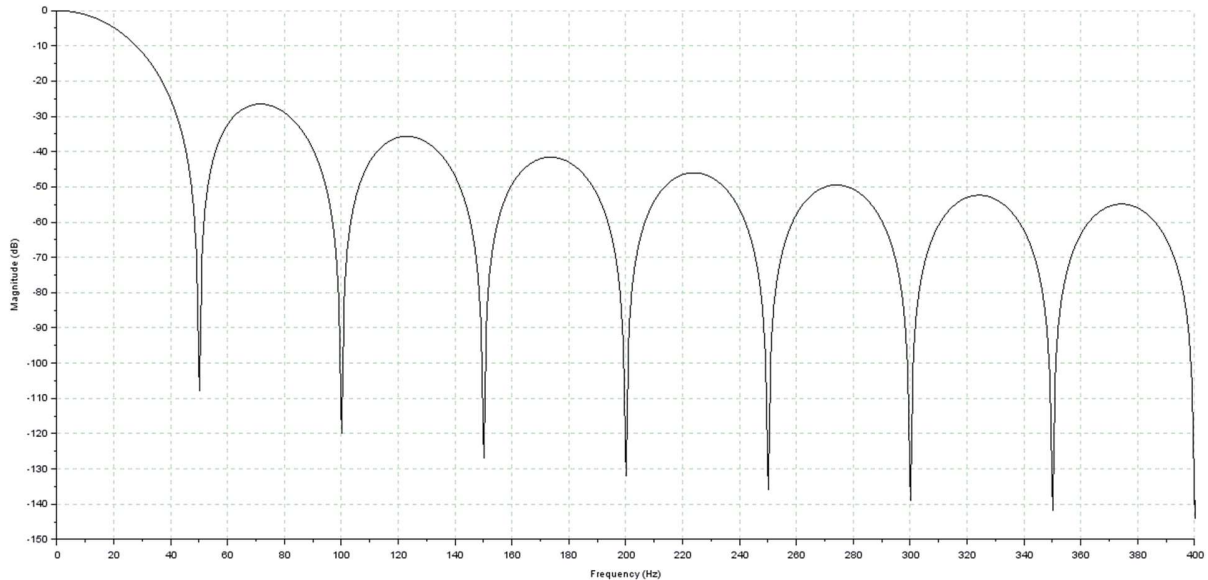
**2.1 Inter-harmonics**

Inter-harmonics are those spectral components of voltages & currents whose frequencies are not integral multiples of the fundamental frequency of the system. The inter-harmonics with frequencies below the fundamental frequency (50 or 60 Hz) are categorized as sub-harmonics. The IEC 61000-4-30 [3] & the IEC 61000-4-7 [4] refer to inter-harmonics as spectral components with frequencies in between two harmonic frequencies.

Some common sources of inter-harmonics include –

1. Furnaces
2. Cyclic loads
3. Cycloconverters
4. Variable speed drives
5. Wind generators

The P class filter attenuates only those harmonic components which are above the nominal frequency value of 50/60 Hz. Those frequencies below the nominal value pass through the filter with minimal attenuation. This can be observed in the magnitude response plot in fig. 1.



**Fig.1:** Magnitude response of the IEEE P class reference filter

The impact of this on the phasor measurements will be discussed in section 3.

## 2.2 Oscillations

Oscillations are fluctuations in the magnitude & frequency of voltages and currents in the power system. In the absence of sufficient damping in the power system, these fluctuations can cause harm to equipment & even result in blackouts. Oscillations can be broadly categorized based on their frequency –

| Frequency         | Oscillation Category      |
|-------------------|---------------------------|
| 0.002 Hz – 0.1 Hz | Very Low Frequency (VLFO) |
| 0.1 Hz – 4 Hz     | Low Frequency (LFO)       |
| 4 Hz – 46 Hz      | Sub Synchronous (SSO)     |

The source of the oscillation determines the frequency & some common ones are generator speed governor systems, control systems, generator natural mechanical frequency interactions, electrical resonant frequency interactions due series compensation, power electronic converters, etc.

Oscillations can be modelled using the principles of amplitude & frequency modulation & the IEEE/IEC 60255-118-1 requires their accurate reporting up to a max oscillation frequency of 2 Hz.

| Class P   |         |
|---|---------|
| Range   | Max TVE |
| Modulation frequency 0.1 to lesser of $F_s/10$ Hz or 2 Hz | 3%      |
|   | 3%      |

This covers only a small portion of the overall oscillation frequency range [5] & the error levels can be significantly higher in the upper end as the sections below will illustrate.

## 3 Influence of sub-harmonic frequencies on phasor measurements

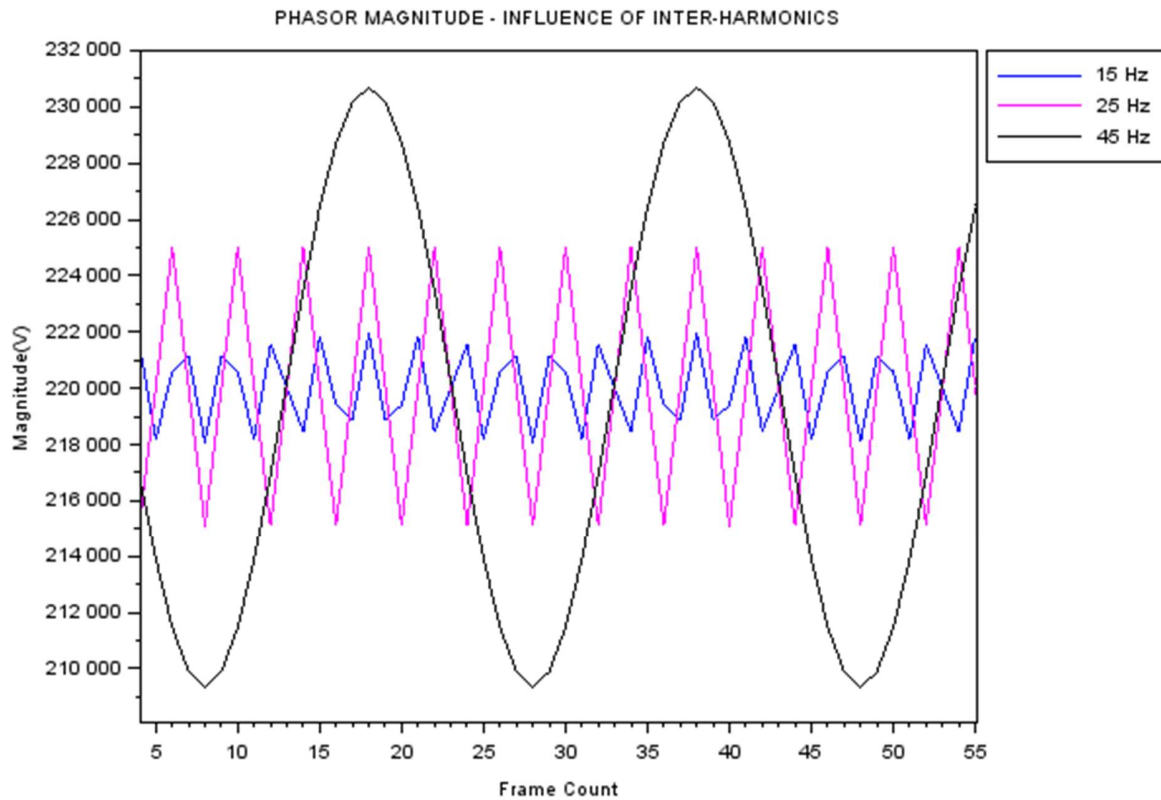
Phasor measurements obtained in the presence of sub-harmonic frequency signals can exhibit several undesirable effects. The influence of such signals is analysed in the following sub-sections for each type of measurement –

1. Phasor magnitude
2. Phase angle
3. Frequency
4. Rate of change of frequency

As reference, a comparison is presented using measurements obtained via an Expert Analysis System which uses advanced signal processing techniques to analyse such system anomalies.

### 3.1 Phasor magnitude

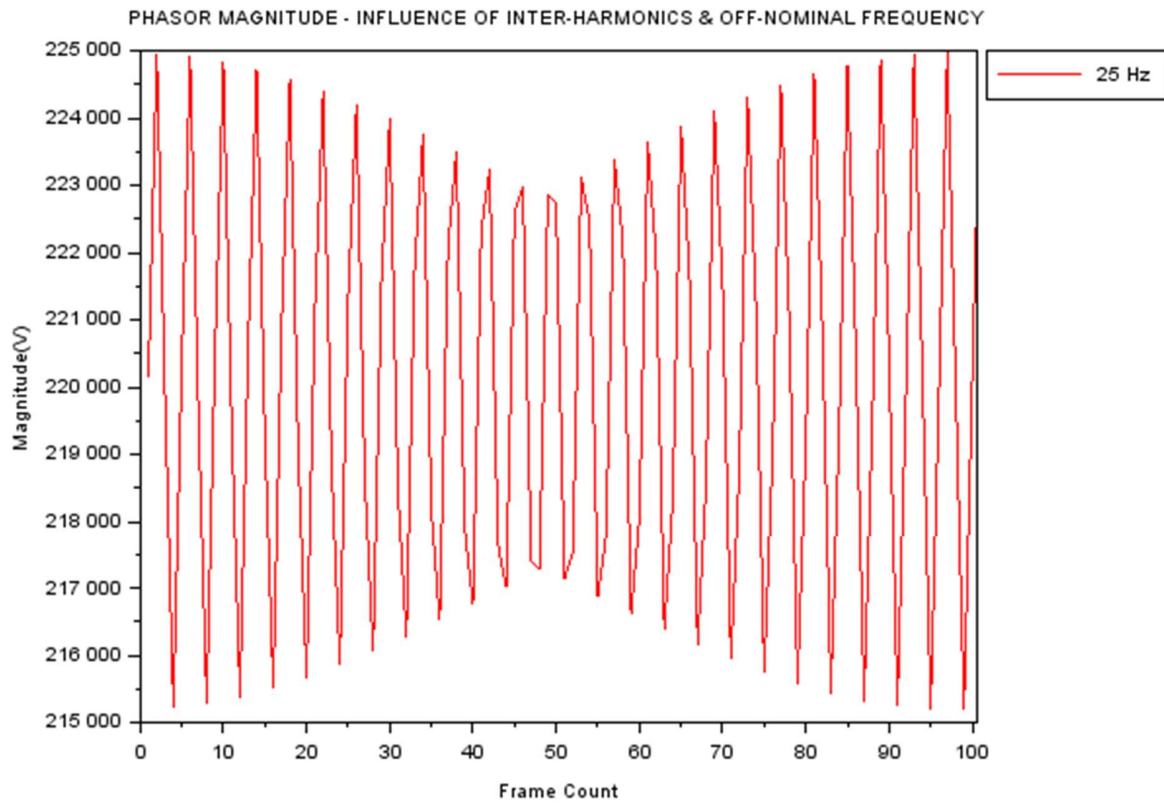
Fig.2 shows the plot of the phasor magnitude estimated in the presence of inter-harmonic signals as per the IEEE/IEC 60255-118-1 P class reference algorithm.



**Fig.2:** Phasor magnitude under the influence of inter-harmonics (15 Hz, 25 Hz & 45 Hz)

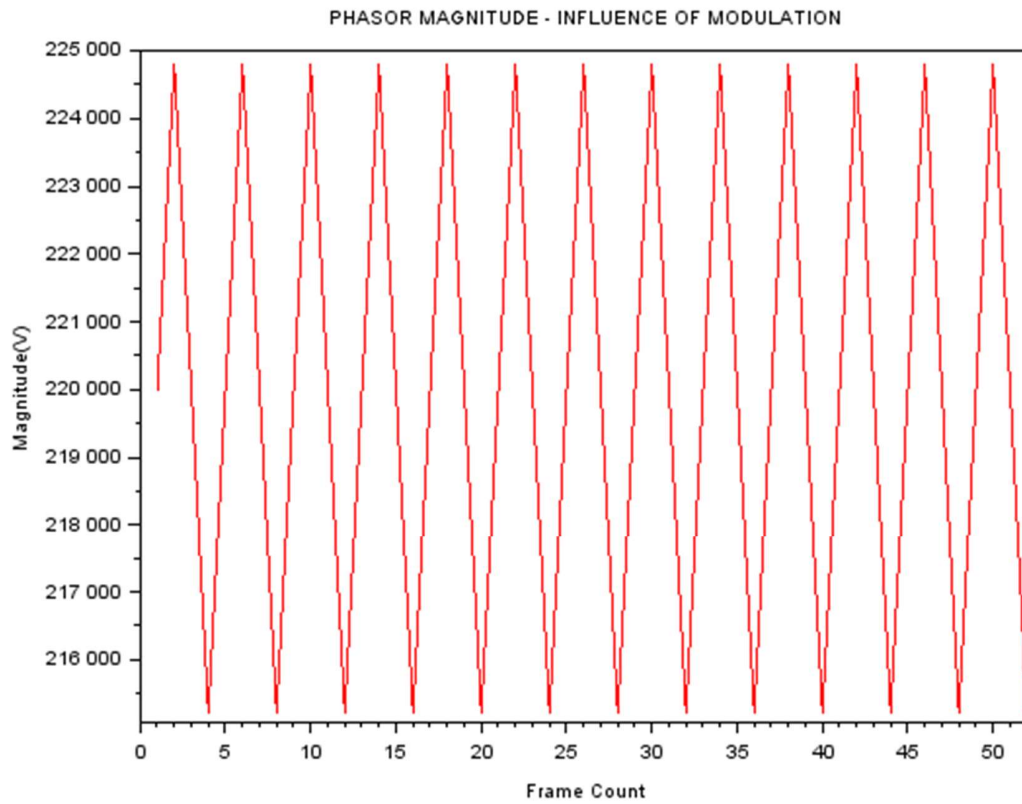
It can be seen that the magnitude oscillates more as the applied inter-harmonic frequency gets closer to the fundamental value. This behaviour is undesirable as it suggests that the oscillations are in the fundamental 50/60 Hz component itself, which isn't the case. Rather, the oscillatory pattern in the magnitude is due to the P class filter which doesn't attenuate inter-harmonics below 50/60 Hz to a sufficient degree.

The measurements shown in fig.3 correspond to the scenario where the fundamental frequency of the system is not the same as the nominal value, i.e. 50/60 Hz. In this example, the frequency is set to 50.5 Hz & the phasor magnitude values exhibit similar oscillatory behaviour but with even more fluctuations. This can be attributed to the fixed frequency filter employed by the P class reference algorithm due to which the filtering & phasor estimation mechanism is unable to track the actual frequency of the system & instead relies on a compensation technique [2].



**Fig.3:** Phasor magnitude under off-nominal frequency condition with presence of inter-harmonics

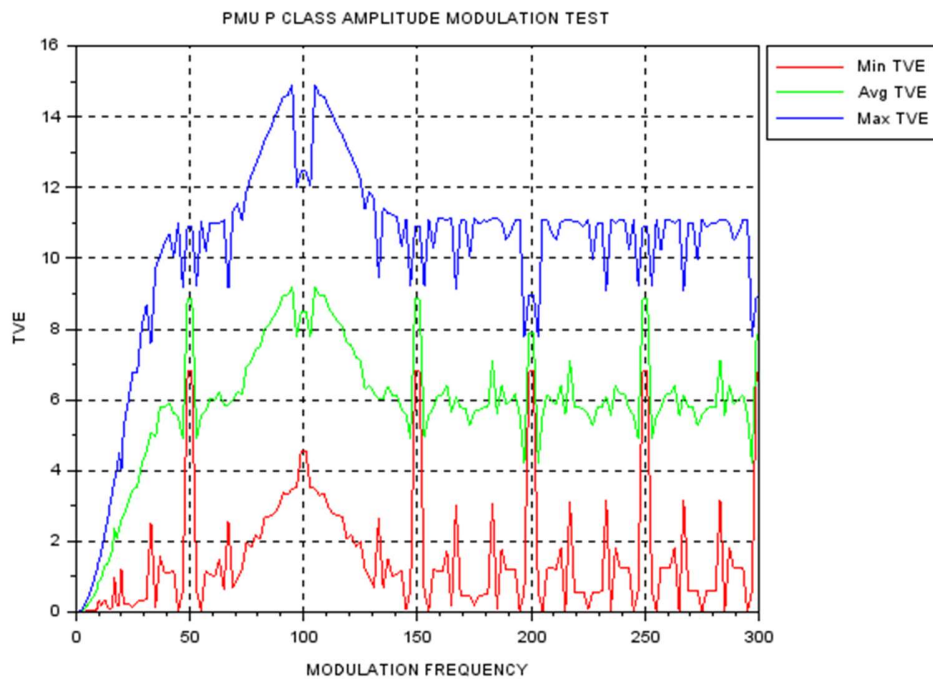
Measurements captured in the presence of a 25 Hz amplitude modulated signal are shown in fig.4. The modulation emulates oscillations in the voltage/current signals & it can be seen that the oscillatory pattern is virtually indistinguishable from the condition where a 25 Hz inter-harmonic is present.



**Fig.4:** Phasor magnitude under the influence a 5% 25 Hz modulating signal

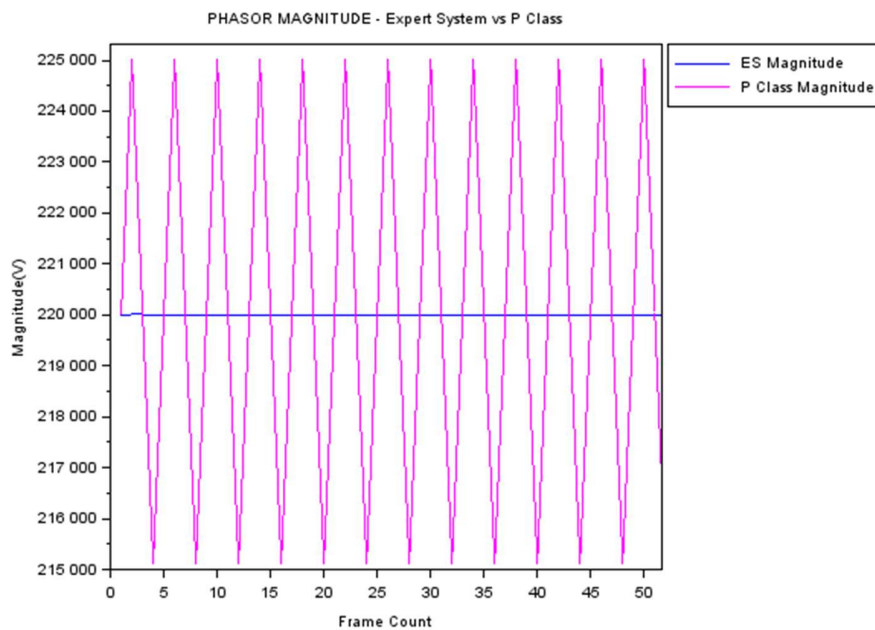
It's also worth noting that the Total Vector Error (TVE) is significantly higher in the presence of modulating signals whose frequency is closer to the nominal value of 50/60 Hz. Oscillatory components at higher frequencies are commonly associated with sub-synchronous resonance (SSR) & fig.5 shows a plot of the TVE vs modulation frequency.

These TVE values are obtained purely under the presence of amplitude modulation restricted to the levels as specified by the IEEE/IEC 60255-118-1 standard. In a real-world scenario, several anomalies might occur simultaneously & the TVE values are likely to be more than what's seen here.

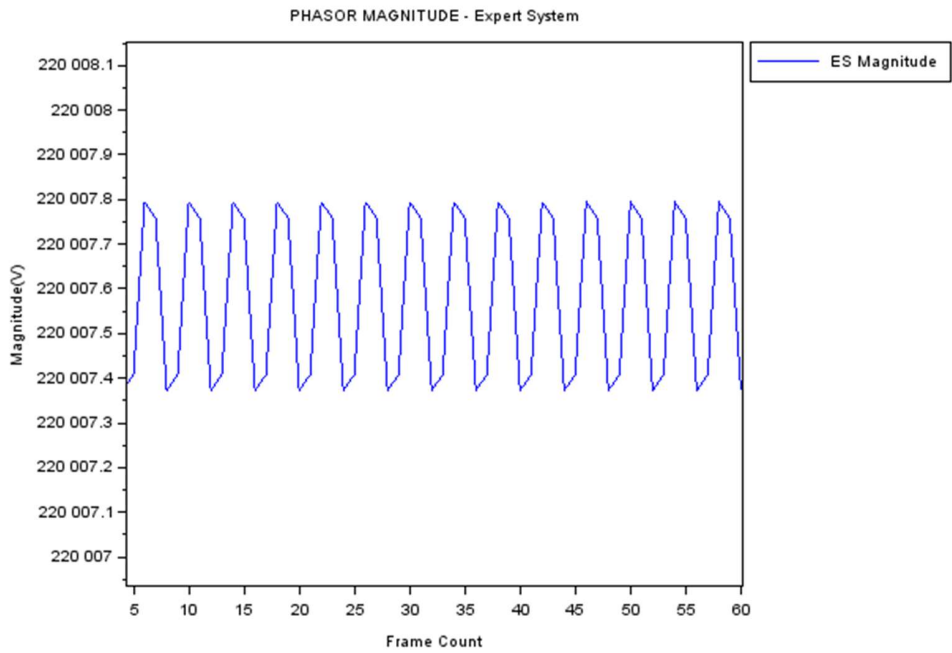


**Fig.5:** Modulation frequency vs TVE for the IEEE P Class reference filter

Lastly, the measurements obtained via the expert analysis system for the 25 Hz inter-harmonic case are shown as reference in fig.6 & fig.7 to highlight the difference when advanced signal processing techniques like frequency tracking, adaptive windowing, etc. are utilized.



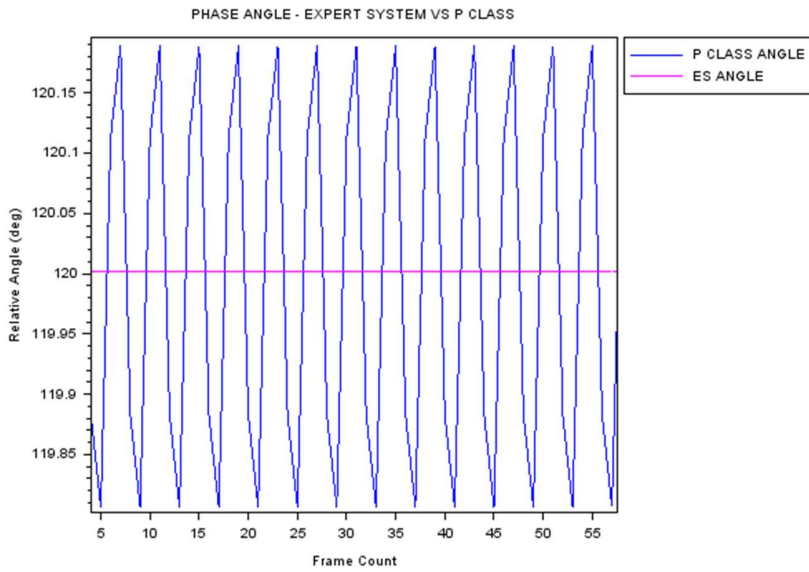
**Fig.6:** Comparison of magnitude estimated by the Expert System analysis & P Class reference design



**Fig.7:** Magnitude estimated by the Expert System analysis zoomed to highlight error levels

### 3.2 Phase angle

The phase angle plot in fig.8 shows a small but consistent oscillation in the angle measurement. Like in the case of the phasor magnitude, this can be attributed to the P class filter response which doesn't attenuate inter-harmonics sufficiently.

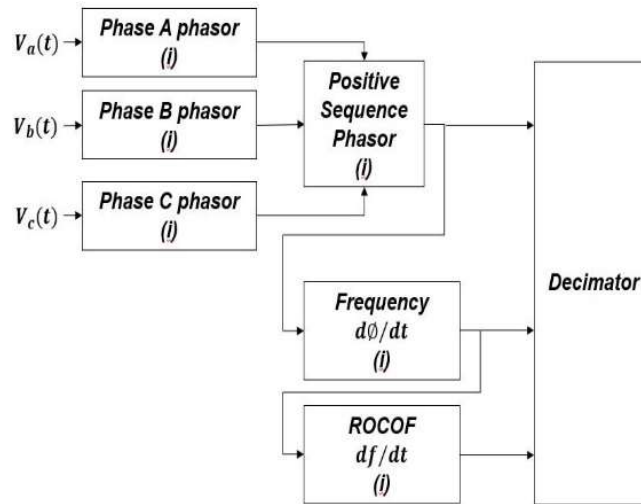


**Fig.8:** Phase angle under influence of inter-harmonics



### 3.3 Frequency

Frequency measurement as per the IEEE 60255-118-1 is done using the rate of change of phase angle.



**Fig.9:** Frequency & ROCOF estimation algorithm as per the IEEE reference model

$$f = f_0 + [\theta(i + 1) - \theta(i - 1)]/[4\pi * \Delta t] \quad (1)$$

where,

$f$  = Estimated frequency

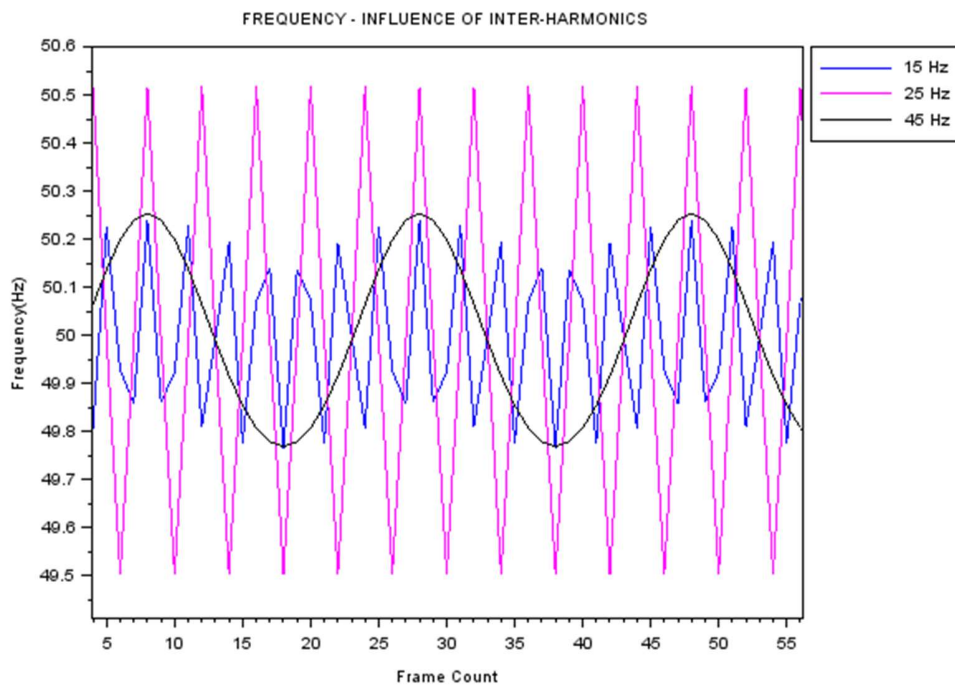
$f_0$  = Nominal frequency (50 Hz or 60 Hz)

$\theta(i + 1)$  = Angle of the  $i + 1^{\text{th}}$  phasor, i.e. the angle following the  $i^{\text{th}}$  estimate.

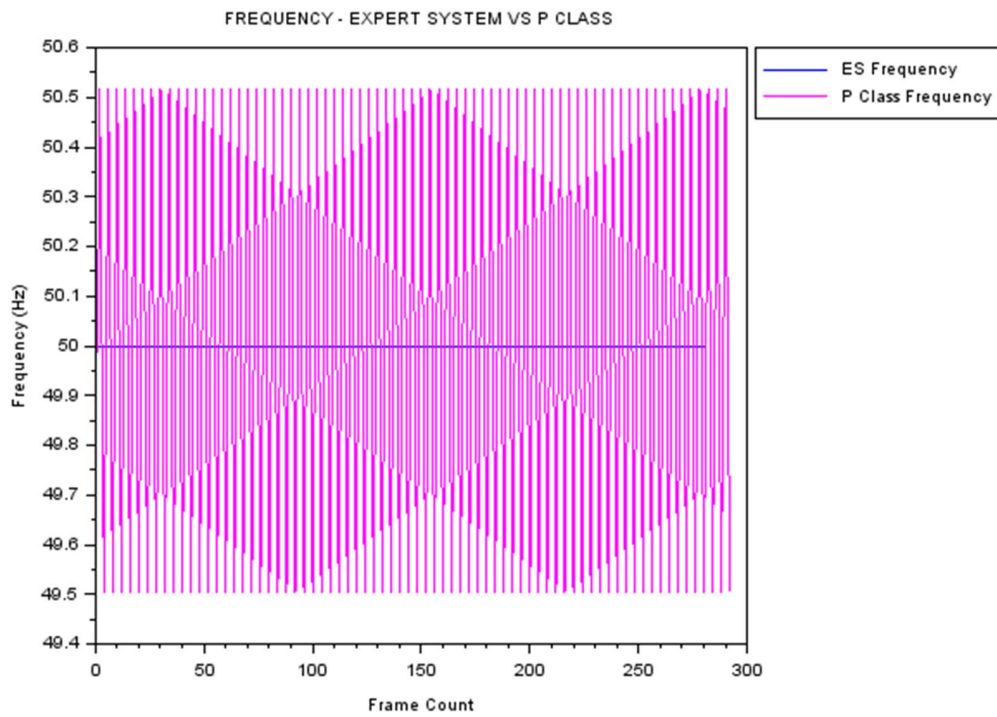
$\theta(i - 1)$  = Angle of the  $i - 1^{\text{th}}$  phasor, i.e. the angle prior to the  $i^{\text{th}}$  estimate

$\Delta t$  = Sampling period

This is the equivalent of differentiation due to which errors in the phase angle are amplified in the frequency measurement. Section 3.2 highlighted the minute oscillations in the phase angle in the presence of inter-harmonics. It can be seen in fig.10 that this small oscillation is translated to a much larger one in the estimated frequency. Even 1% of the 25/30 Hz inter-harmonic can introduce fluctuations as high as +/- 100 mHz.



**Fig.10:** Frequency estimated in presence of inter-harmonics (15 Hz, 25 Hz & 45 Hz)



**Fig.11:** Comparison of frequency estimated by the Expert System analysis & P Class reference design

### 3.4 Rate of change of frequency (ROCOF)

Rate of change of frequency is estimated using a derivative of the rate of change of phase angle.

$$df = [\theta(i + 1) + \theta(i - 1) - 2\theta(i)]/[2\pi * \Delta t^2] \quad (2)$$

where,

$df$  = Estimated ROCOF

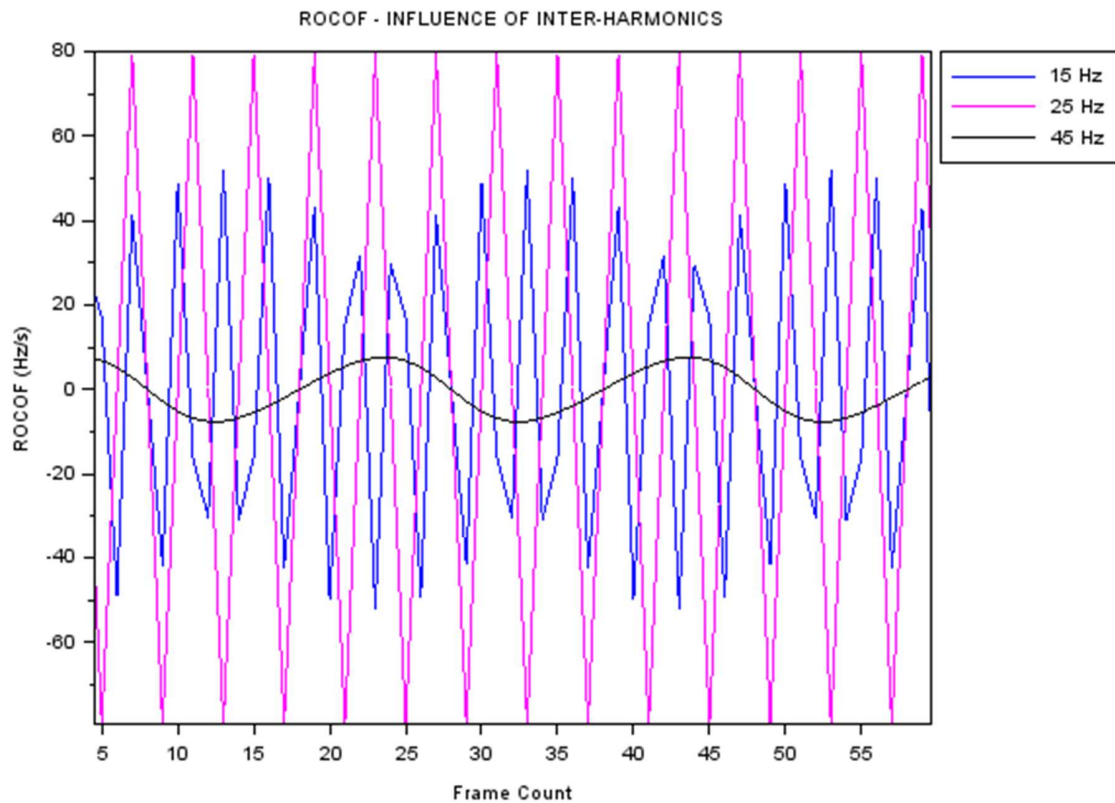
$\theta(i)$  = Angle of the  $i^{\text{th}}$  phasor.

$\theta(i + 1)$  = Angle of the  $i + 1^{\text{th}}$  phasor, i.e. the angle following the  $i^{\text{th}}$  estimate.

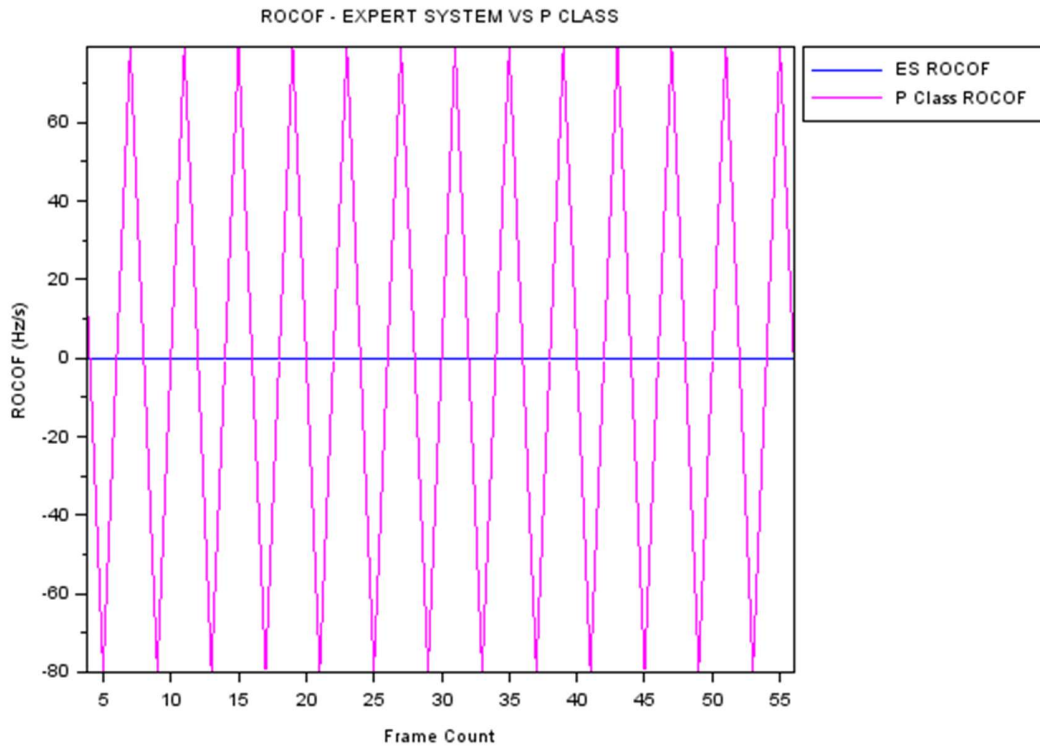
$\theta(i - 1)$  = Angle of the  $i - 1^{\text{th}}$  phasor, i.e. the angle prior to the  $i^{\text{th}}$  estimate

$\Delta t$  = Sampling period

It should be no surprise that the ROCOF value is seen to oscillate by a significant amount considering the variations in frequency & phase angle observed in sections 3.2 & 3.3. The degree of error simply makes the ROCOF value unusable.



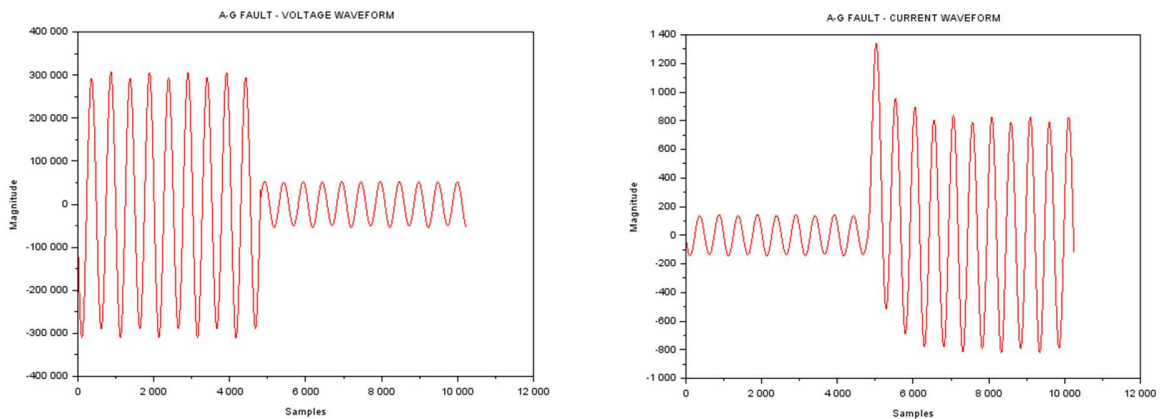
**Fig.12:** ROCOF estimated in presence of inter-harmonics (15 Hz, 25 Hz & 45 Hz)



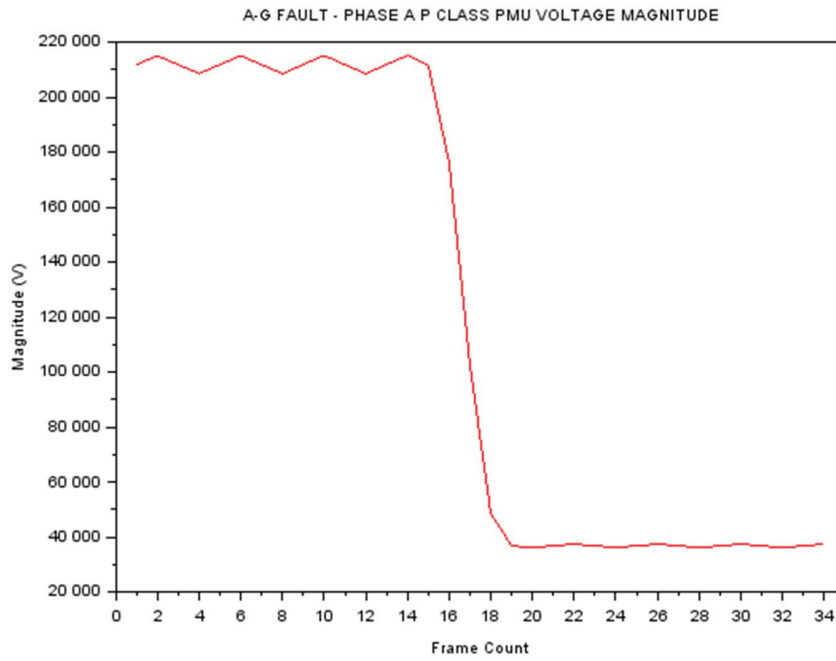
**Fig.13:** Comparison of ROCOF estimated by the Expert System analysis & P Class reference design

### 3.5 System disturbances

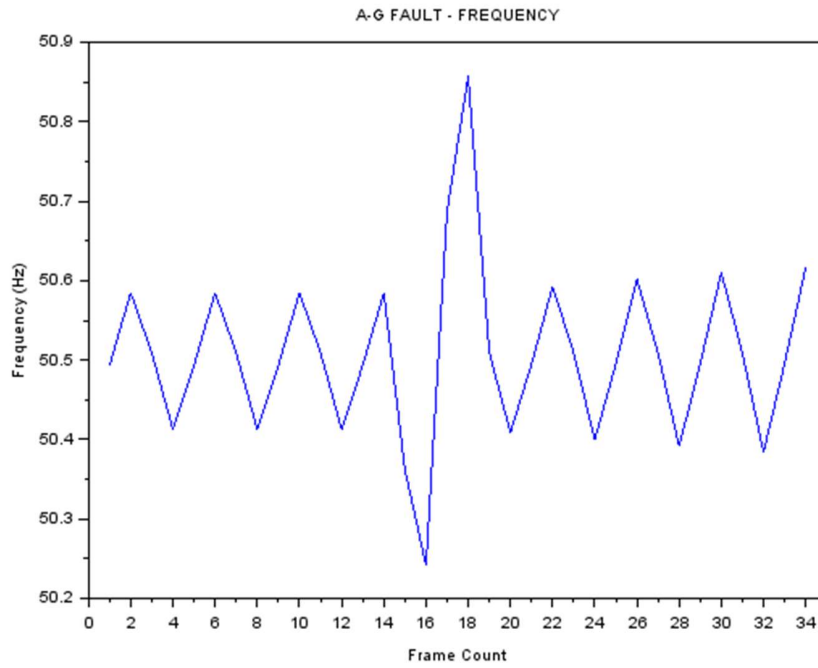
In a typical power system, it's more likely that several system anomalies occur at the same time in the event of a disturbance. Under such circumstances, the errors described above would have a combined effect on the quality & accuracy of the measurements. The following is an example of such an event – a simulated phase A to ground fault on a 220 kV line with 1% inter-harmonic at 25.25Hz, 5% oscillation with a frequency of 25Hz & an off-nominal fundamental frequency of 50.5 Hz –



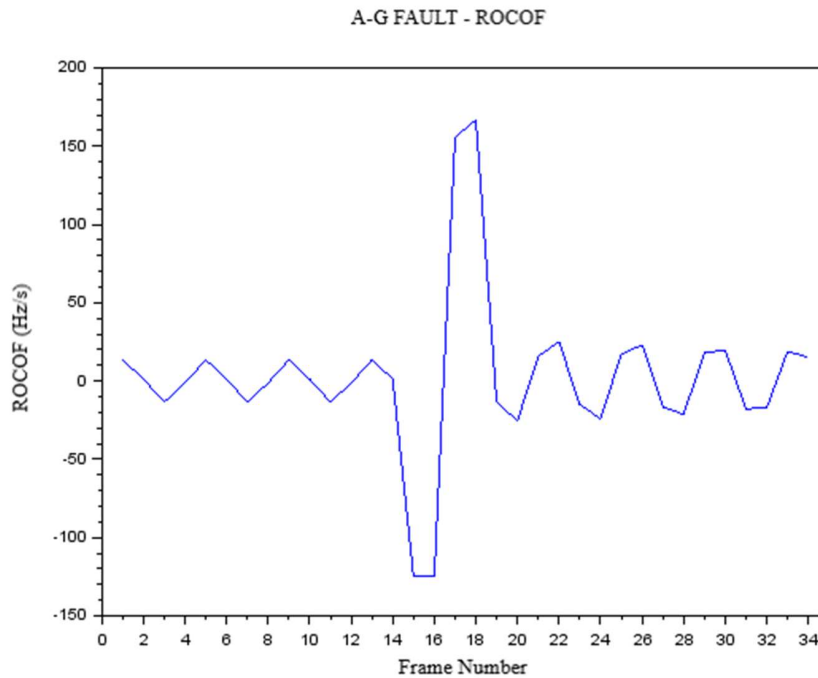
**Fig.14:** A-G fault voltage & current waveforms



**Fig.15:** A-G fault phase A magnitude estimated using the P class reference model



**Fig.16:** A-G fault frequency estimated using the P class reference model



**Fig.17:** A-G fault ROCOF estimated using the P class reference model

The combined effect of these anomalies can be seen clearly in the various measurements which exhibit considerable errors. This is in line with the observations in previous sections & the errors are attributed to the lack of inter-harmonic filtering capabilities of the P class reference filter. It is also worth noting that the individual contributions of the 25 Hz inter-harmonic and the 25 Hz amplitude modulation to the oscillatory pattern in the phasor magnitude cannot be determined. As such, the magnitude oscillations are more than expected due to the contribution of the 25 Hz inter-harmonic which could have been eliminated via relevant signal processing techniques.

#### 4 Conclusion

The observations in this paper highlight the challenges associated with the usage of synchrophasor measurements collected in the presence of sub-harmonic frequencies. The following considerations can be made based on these results when using this data for analytical applications –

1. The IEEE P class reference model phasors offer limited immunity to sub-harmonic frequencies.
2. The choice of the filter & fixed frequency design methodology are the key sources for the anomalies in the measurements.
3. The use of frequency tracking methods can provide better levels of accuracy under off-nominal frequency conditions. This technique can also be extended to work under real-time conditions to be compliant with the IEEE/IEC 60255-118-1 [6].
4. Choice of filter plays an important role in the suppression of inter-harmonic signals which at frequencies close to the nominal value can introduce significant errors as well as overlap with oscillations.
5. With offline techniques, it's possible to employ advanced signal processing methods such as intelligent window selection which allows the phasor estimator to dynamically adjust the window to obtain fast response during transients & high accuracy in their absence.

6. Oscillation monitoring, especially those of higher frequencies, should be done using signal processing methods that're dedicated for this purpose rather than relying on phasor measurements which may suffer from accuracy issues in the presence of system anomalies.
7. Use of the more recent Point-on-Wave (PoW) data, also known as Wave Measurement Unit (WMU), along with PDCs having the capabilities mentioned above could be an alternative solution. With the availability of waveform data, it's possible to perform analysis targeted for a certain application.

Exploration of the feasibility of using a frequency tracking mechanism & analysis on the choice of the filter in future revisions of the IEEE/IEC 60255-118-1 standard might also be a worthwhile exercise.

## References

- [1] IEEE Standard for Synchrophasor Measurements for Power Systems, IEEE Standard C37.118.1-2011/2014, 2011/2014
- [2] IEEE/IEC International Standard - Measuring relays and protection equipment - Part 118-1: Synchrophasor for power systems – Measurements, IEEE/IEC 60255-118-1, 2018
- [3] Electromagnetic compatibility (EMC) – Part 4-30: Testing and measurement techniques – Power quality measurement methods, IEC 61000-4-30, 2015
- [4] Electromagnetic compatibility (EMC) – Part 4-7: Testing and measurement techniques – General guide on harmonics and interharmonics measurements and instrumentation, for power supply systems and equipment connected thereto, IEC 61000-4-7, 2002
- [5] VISOR: Close Down Report, 2018
- [6] Andrew J. Roscoe, "Exploring the Relative Performance of Frequency-Tracking and Fixed-Filter Phasor Measurement Unit Algorithms Under C37.118 Test Procedures, the Effects of Interharmonics, and Initial Attempts at Merging P-Class Response With M-Class Filtering", IEEE Transactions on Instrumentation and Measurement, Vol. 62, No. 8, August 2013