

Frequency Monitoring Network (FNET) and its Aspects on Disturbance Triggering and Disturbance Location Estimation

Tao Xia Yilu Liu Robert M. Orndorff

Abstract-- The Internet Based real-time GPS synchronized wide-area Frequency Monitoring Network (FNET) is an extremely low cost and quickly deployable wide-area frequency measurement system with high dynamic accuracy which consists of Frequency Disturbance Recorder (FDR) distributed to more than 60 places around North America and an Information Management System situated at Virginia Tech. Since its first FDR deployment in 2003, the FNET system has been proved to be able to reliably receive frequency data accurately measured at and instantaneously sent from different locations of interest via the Internet, and efficiently run the analyzing program to detect and record significant system frequency disturbances and subsequently estimate the location of disturbance center, namely the event location, in the electric grid based on the information gathered. The excellent performance of the FNET system so far has made power grid situation awareness monitoring based on distribution level frequency measurements a reality, and thus advances our understanding of power system dynamics to a higher level and in a broader dimensionality. The first part of this paper briefly introduces the genesis and the architecture of the FNET system, followed by the summarization of its concrete implementations. After that, the paper outlines the disturbance triggering methods to detect the frequency disturbance when a generator trip or a load rejection occurs in the electric grid, which include the frequency deviation method and the rate of change of frequency threshold method. Lastly, the paper discusses the algorithm developed at FNET to estimate the disturbance location by the triangulation procedure using real-time frequency data and geographic topology of the FNET units in the power grid where the disturbance happens. The data processing, data plotting, and event display methods, as well as a disturbance size estimation module will also be mentioned, and a number of example cases will be adduced.

Index Terms-- Frequency measurement, Frequency Monitoring Network (FNET), Event location estimation, Event triangulation.

I. THE GENESIS AND THE ARCHITECTURE OF THE FNET SYSTEM

SYSTEM frequency is one of the most important parameters of the power system. Area frequency can vary over a small range due to generation-load mismatches. Many power system protection and control applications require accurate and fast estimation of area system frequency. Frequency information has also been used routinely in the

Tao Xia, currently a PhD student in the Bradley Department of Electrical and Computer Engineering at Virginia Tech, E-mail: xiat@vt.edu Phone: 540-383-5091

Yilu Liu, professor of Electrical Engineering at Virginia Tech and IEEE Fellow, E-mail: yilu@vt.edu Phone: 540-231--3393

Robert M. Orndorff, System Operations - Fault Analysis Department, Dominion Virginia Power, E-mail: Robert.M.Orndorff@Dom.com Phone: 804-257-4960

control and management of generations. Fast and accurate real time measurement of frequency is an important component in the operation of the transmission system. Moreover, if the rate of change of frequency is known, more accurate adjustments can be made to restore the system back to its normal operation point quickly.[1]

Over the years, many advances had been made in the areas of accurate and efficient frequency information retrieval, and many measurement devices had been researched and brought up by power system engineers, most of which, however, had assumed a single system frequency and accordingly used long periods of data averaging in order to achieve good estimation accuracy until the PMU came into its very being. In the early 1990s the synchronized phasor measurement unit (PMU) was developed and later commercialized. Since then, a number of applications have been proposed that require wide-area measurement systems. The results of all earlier efforts clearly point to the need for much wider measurement coverage, coverage that can be quickly and economically obtained.[2]

The objective of the frequency monitoring network (FNET) effort is to create an extremely low cost and quickly deployable wide-area frequency measurement system with high dynamic accuracy and that requires a minimal installation cost. All these features are possible in FNET due to the fact that the power system frequency can be accurately measured and global positioning system (GPS) synchronized at the 110-V distribution voltage level of a typical office outlet.[1,3,4]

The FNET system is a wide-area sensor network consisting of high-precision frequency disturbance recorders and a central processing server. Fig.1 shows the framework of the FNET system.

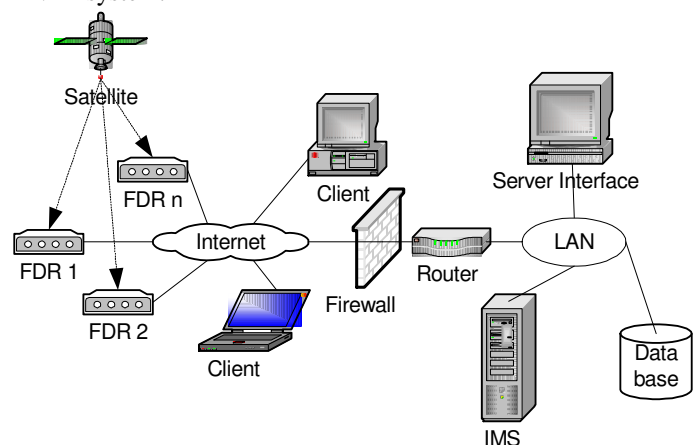


Fig. 1. Frequency Monitoring Network System Architecture

The FNET system consists of two major components: a)

Frequency Disturbance Recorders (FDRs), which perform local GPS synchronized frequency measurements and send data to a server through the Internet; b) The Information Management System (IMS) which includes data collection and storage service, data communication service, database operation service, and web service. The FDRs sample the voltage at various locations; calculate the frequency, angle, and magnitudes of the voltage; timestamp the results using GPS-based synchronized timing; and send back the data to the IMS through the Internet. The IMS receives, classifies, and manages the data. The disturbance detecting trigger is embedded in the IMS and based on the data stored in the database. Furthermore, the IMS estimates the event location and then produces the corresponding event map after detecting a disturbance. The result is then uploaded to the website for easy reference.

The entire FNET program consists of four individual modules including the FNET main server (FNETServer.exe), the event plotting module (EventPlot.exe), the triangulation module (Triangulation.exe), and the event map generation module (GenMap.exe), shown in Fig. 2.

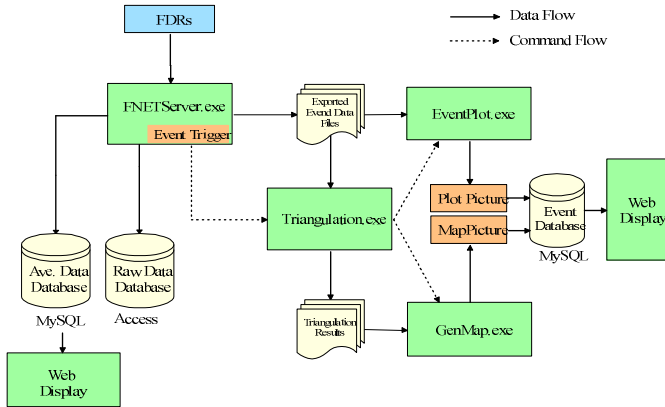


Fig. 2. FNET Software Framework

The FNET main server acts as the IMS that collects the frequency data from the internet, saves the data to both a Microsoft[®] Access database and a MySQL[®] database, and checks for events. An on-line event trigger is also implemented in the server. When an event is detected on-line, server exports event data to hard disk files; records the event date, time, and zone information into EventTime.txt; and then calls the triangulation module.

The triangulation module, which is also referred to as the event location estimation module, reads in data from exported data files, performs data conditioning, and estimates the event location. After the event location is estimated, the module writes the triangulation results to a text file and then calls both the event plotting module and the event map generation module. The event plotting module plots the exported event frequency data and saves it to a file for web display. The event map generation module displays the event's location coordinates on a U.S. map along with information regarding the event type and estimated trip amount.

II. FNET IMPLEMENTATION

A. Frequency Disturbance Recorder

An FDR unit consists of a voltage transducer, a low pass filter, an analog to digital (A/D) converter, a GPS receiver, a microprocessor, and the network communication modules. The voltage transducer takes an analog voltage signal from an 110V wall outlet and converts it to acceptable A/D levels, the low pass filter eliminates the high frequency components, and the A/D converter transforms the analog signal into digital data. A microprocessor is used to generate the sampling pulses synchronized to the 1pps from the GPS receiver integrated into the FDR. The phase angle, frequency, and rate of change of frequency are computed, using phasor techniques developed specifically for single phase measurements. The computed values are time stamped, and transferred to the IMS via the Internet. There are two generations of FDRs, which are shown in Fig. 3. The right one is generation I and the left one generation II.

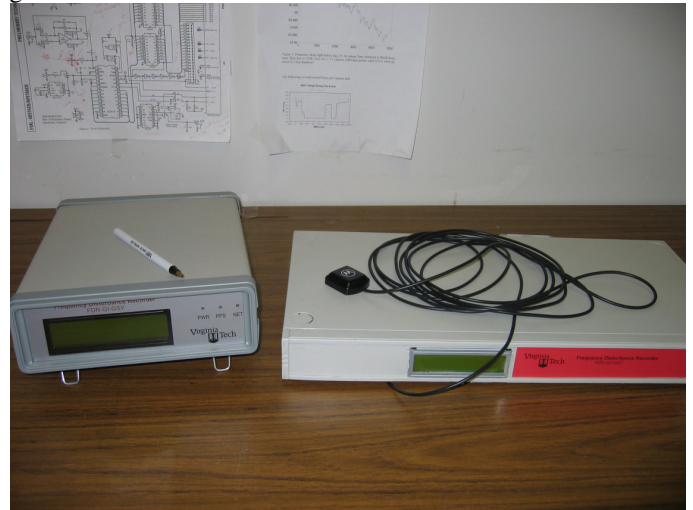


Fig. 3. Frequency Disturbance Recorders

Static frequency computations are usually done on voltage waveforms due to their relatively unchanging nature during normal operating conditions. The FDR unit makes frequency calculations using algorithms of phasor analysis and signal resampling techniques. The complexity of the calculations is minimized to allow the microprocessor time to complete its other tasks and prevent data overflow. The current version of FDRs has a sampling rate of 1440Hz and the resulting frequency accuracy is $\pm 0.0005\text{Hz}$ or better.

B. Frequency Disturbance Recorder Placement

The FDR units are designed to record dynamic frequency information for power system analysis (and control in the future). The placement of FDR units is an important issue for the FNET system. Their location should be selected to effectively reflect the different frequency clusters of inter-area oscillations and to cover as broad an area as possible, in order to capture dynamic behavior of larger system disturbance. Just like PMU placement, FDR locations should represent the system frequency, effectively describe the behavior of major inter-connected systems, and provide information on the large area load behavior. The present placement of FDRs is aimed to

cover all the regional reliability regions that form North American Electric Reliability Council (NERC). In the future FNET system, more FDR units will be located close to major generation centers, major transmission tie lines and load concentrated areas for local frequency oscillation mode study. Up to now there have been approximately 50 measurement devices deployed all around the North America. Fig. 4 shows the locations of the units that are deployed in the FNET system. Thanks to their minimal installation cost and plug and play operation, FDR units can be easily relocated, if necessary.

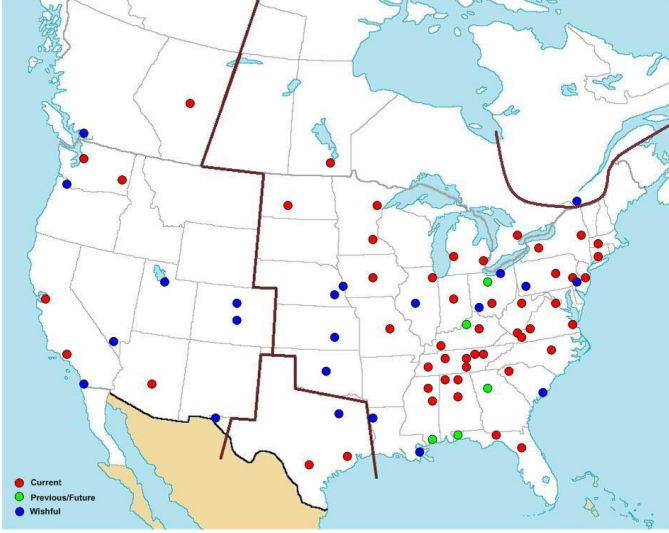


Fig. 4. FDR Distribution Map

C. FNET Server Hardware and Software Configuration

The server receives data from different FDR units, processes the data, manages the database, performs data analysis, and supports the web service for the Internet users. The frequency IMS server can be physically located anywhere. The server has following system configurations:

- Hardware configuration: Two quad CPU (Intel Xeon CPU 3.0GHz), 16GB memory servers with 6TB hard disks are employed to meet the need of reliable data transmission, processing and web service. The backup data is stored both on the hard disk of the server and in an external hard drive.
- Server software configuration: Windows Server 2003 is the host operating system. A commercial database is installed at the database server machine, as the backend database. C++ is employed for the whole server program design and communication. Internet standard web browsers are used as the interface for the user to access the frequency data.

The web service that the FNET servers host includes the frequency display and the historical event database browsing functions, and it provides a common easily accessible user interface to observe the frequency information and event database from anywhere on the Internet. The functionality of the frequency display is only limited to viewing the average frequency information of the system being monitored. The server pages are written in PHP and Javascript.

More information about FNET is available at the website www.FNETpublic.ece.vt.edu.

III. DISTURBANCE TRIGGER

When a generator trip or a load rejection occurs in the electric grid, the imbalance of generation and load causes sudden frequency changes within the system. Therefore, the frequency's rate of change can be used as an indicator for grid disturbances as mentioned above. Likewise, the difference between pre- and post-disturbance frequency averages can also be utilized as a criterion to distinguish the abnormality of the event condition and normal operation condition. The estimated amount of tripped generation or load shedding is based on the relationship between frequency change and active power balance.

The frequency deviation method is illustrated in Fig. 5, where it shows the method continuously calculates the frequency average of the time period T3 that is immediately prior to the current time, and the frequency average of the time period T1 that is T2+T3 interval before the current time, recording them as f_{post} and f_{pre} respectively. The algorithm then proceeds to calculate the frequency difference between the two averages as Δf . If the frequency difference Δf exceeds a predetermined threshold, then an event has occurred. The time periods T1 and T3 are to reduce the noise to an acceptable amount; the time period T2 is to cover the frequency-change time of an event in order for the frequency averages to faithfully reflect the pre- and post-disturbance conditions.

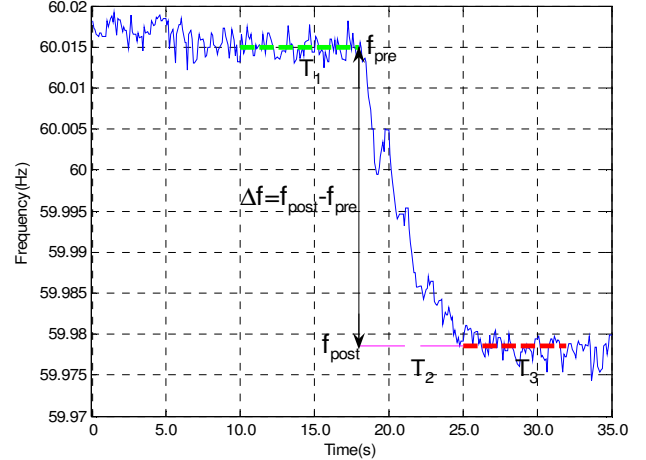


Fig. 5. Schematic Diagram of Frequency Deviation Method

A chart explaining the rate of change of frequency event trigger algorithm is shown in Fig. 6. This example assumes the use of two FDRs. However, in general, a trigger can be created using data from n FDRs. (Note: It is important to understand that each of the n FDRs should be connected to the same electrical interconnection.) This on-line event trigger algorithm can be described as follows:

1. Perform data conditioning using a moving median filter or other data smoothing technique and save the data in a buffer.

2. Calculate $\frac{df}{dt_1}, \frac{df}{dt_2}, \dots, \frac{df}{dt_n}$ where:

$$\frac{df}{dt_i} \equiv \frac{\Delta f_i}{\Delta t_i} \quad (1)$$

3. If:

$$\left| \frac{df}{dt_1} \right|, \left| \frac{df}{dt_2} \right|, \dots, \left| \frac{df}{dt_n} \right| \geq \tau \quad (2)$$

where τ is an empirically determined threshold and n is the number of FDRs used, then the software triggers on the event.

This event trigger is applied continuously to incoming frequency data as it arrives at the IMS. The trigger allows for the capturing of frequency excursions that are further used for the location of the hypocenter of the inciting event.

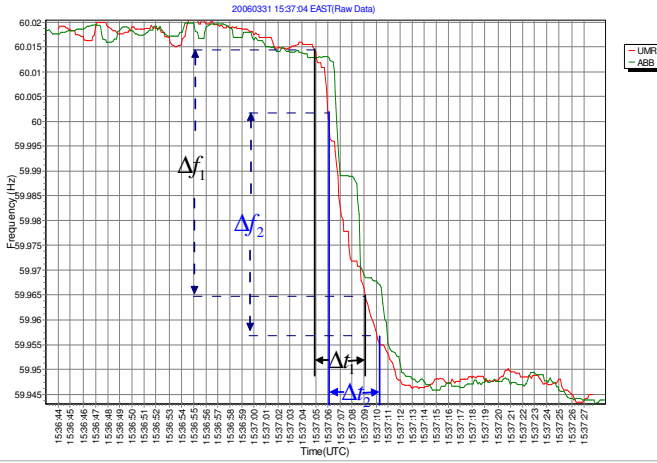


Fig. 6. Schematic diagram of event trigger algorithm

The amount of tripped generation is proportional to the frequency drop, and can be calculated by the area frequency response characteristic β and the frequency change.

When an electrical load change occurs, the turbine-generator rotor accelerates or decelerates, and frequency undergoes a transient disturbance. Under normal operating conditions, the rotor acceleration eventually becomes zero, and the frequency reaches a new steady-state, which is shown in the steady-state frequency-power relation for turbine-governor control:

$$\Delta p_m = \Delta p_{ref} - \frac{1}{R} \Delta f \quad (3)$$

where Δf is the change in frequency, Δp_m is the change in turbine mechanical power output, and Δp_{ref} is the change in a reference power setting. R is called the regulation constant.

The steady-state frequency-power relation for one area of an inter-connected power system can be determined by summing (3) for each turbine-generating unit in the area. Noting that Δf is the same for each unit,

$$\begin{aligned} \Delta p_m &= \Delta p_{m1} + \Delta p_{m2} + \dots \\ &= (\Delta p_{ref1} + \Delta p_{ref2} + \dots) - \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots \right) \Delta f \quad (4) \\ &= \Delta p_{ref} - \frac{1}{R} \Delta f \end{aligned}$$

where Δp_m is the total change in turbine mechanical power output, and Δp_{ref} is the total change in a reference power setting within the area. We define the area frequency response characteristic as $\beta = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots \right)$.

It is apparent that since Δp_{ref} is fixed during the short time period of an event, the Δp_m , also interpreted as the tripped generation, is directly proportional to the change in frequency.

IV. EVENT LOCATION ESTIMATION

A. Wave-front Arrival Time

Frequency perturbations have been shown to travel throughout the grid as electromechanical waves dispersing at finite (measurable) speeds.[5-7] Hence, the FDRs located throughout the grid detect said waves with unique time delays proportional to, among other factors, the physical distance between each respective unit and the disturbance location. We refer to these various detection times as “wave-front arrival times.” For indeed, these times correspond to the “arrival” of an electromechanical wave caused by the inciting event. Thus, the first step in event location is the determination of the wave-front detected by each unit and its corresponding arrival time.

Listed below is the approach that determines the wave-front arrival times using FNET’s FDRs.

First, after the on-line event trigger detects an event, three seconds worth of data is used to calculate the pre-disturbance frequency as shown in (5):

$$f_{pre} = \frac{1}{n} \sum_{i=1}^n \left(\frac{1}{k} \sum_{j=1}^k f_{i,j} \right) \quad (5)$$

where n is the total number of available units, k is the number of the points in three seconds of data, i is the index of units, j is the index of data, $f_{i,j}$ is the corresponding j^{th} frequency measurement of unit i .

Second, a confidence threshold, \mathcal{E} , has been fixed at 0.008 Hz, which is justified empirically by an abundance of previous event cases. Then, the wave-front frequency, the frequency at which we will note the event detection time for each unit, is calculated with f_{pre} and \mathcal{E} . This quantity can be defined as:

$$f_{wavefront} = f_{pre} - \mathcal{E} \text{ or } f_{wavefront} = f_{pre} + \mathcal{E} \quad (6)$$

depending on the type of event at hand. Generator trips in general cause negative frequency excursions and load shedding in general causes positive frequency excursions.

Finally, the detection times are documented as the time when the measured frequency data crosses the line determined by $f_{wavefront}$. A frequency plot illustrating wave-front arrival time is shown in Fig. 7.

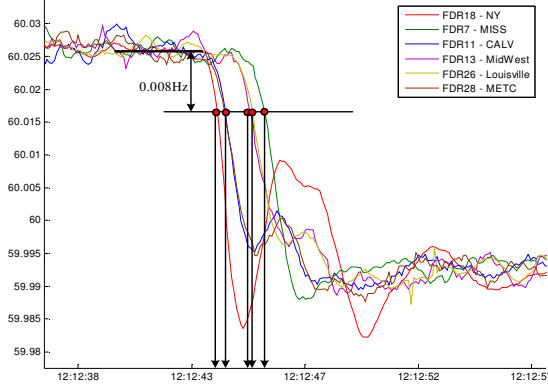


Fig. 7. Example of fixed frequency shift wave-front arrival time detection method

B. Newton's method

When an event occurs, the locations and the wave-front arrival times of the units are known. Event location, (x_e, y_e) – latitude and longitude, are two unknown variables. Event time, t_e , is the third unknown variable. If we say that the wave propagation speed, V , is unique in different directions (as past experience suggests), V will be the fourth unknown vector-valued variable in the event location estimation problem.

Since there are four unknown variables, one of which is vector-valued, V , standard linear solution methods such as least-squares will not work. Here, we discuss how to use Newton's method to do triangulation. There are more than thirty FDRs deployed in the Eastern US. However in this method, we choose the first four units in the response sequence to find event location.

As shown in Fig. 8, assume the event is located at (x_e, y_e) , the event hypocenter, then the distances between event location and an arbitrary FDR location is provided by:

$$L_i^2 = (x_i - x_e)^2 + (y_i - y_e)^2 \quad (7)$$

where L_i is the distance between FDR_i and the event hypocenter, and x_i and y_i are the latitude and longitude of

FDR_i's location respectively.

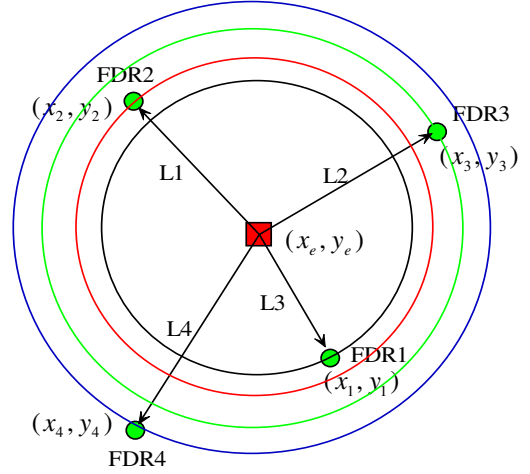


Fig. 8 Chart of fixed frequency shift wave-front arrival time detection method

Distance can also be expressed by the product of speed and time:

$$L_i = V(t_i - t_e) \quad (8)$$

where t_i is the wave-front arrival time determined by information from FDR_i and t_e is actual event time. Note that t_i is always larger than t_e since the event cannot be detected before it occurs. Hence we arrive at the event location estimation equations:

$$\begin{cases} (x_1 - x_e)^2 + (y_1 - y_e)^2 - V^2(t_1 - t_e)^2 = 0 \\ (x_2 - x_e)^2 + (y_2 - y_e)^2 - V^2(t_2 - t_e)^2 = 0 \\ (x_3 - x_e)^2 + (y_3 - y_e)^2 - V^2(t_3 - t_e)^2 = 0 \\ (x_4 - x_e)^2 + (y_4 - y_e)^2 - V^2(t_4 - t_e)^2 = 0 \end{cases} \quad (9)$$

Corresponding initial values of these unknown variables are needed to solve these non-linear equations using Newton's method. The averages of the locations of FDRs are set as the initial values of event location:

$$x_{e0} = (x_1 + x_2 + x_3 + x_4)/4 \quad (10)$$

$$y_{e0} = (y_1 + y_2 + y_3 + y_4)/4 \quad (11)$$

The time initial is set to the time that is 0.5 second before the earliest wave-front arrive time:

$$t_{e0} = t_1 - 0.5 \quad (12)$$

The initial of wave speed is set as 200miles/sec, according to previous experience.

Now we can use Newton's method to calculate the four unknown variables, and get the estimated event location.

C. Least Squares Method

Newton's method discussed above uses only four FDRs' data to compute the event hypocenter, instead of using all of the available information. Moreover, it does not always reach a convergent solution for the set of non-linear equations.

Another triangulation method based on the least squares method is introduced that can use all available FDR information and will almost always give a well conditioned result.

According to (9), for each responding FDR, we can write:

$$\begin{aligned} (x_1 - x_e)^2 + (y_1 - y_e)^2 &= V^2(t_1 - t_e)^2 \\ (x_2 - x_e)^2 + (y_2 - y_e)^2 &= V^2(t_2 - t_e)^2 \\ &\vdots \\ (x_n - x_e)^2 + (y_n - y_e)^2 &= V^2(t_n - t_e)^2 \end{aligned} \quad (13)$$

where (x_n, y_n) are the (x, y) coordinates of the n^{th} FDR to respond; (x_e, y_e) are the (x, y) coordinates of the hypocenter; t_n is the time at which the n^{th} FDR measures the electromechanical wave; t_e is the time at which the event occurred; and V is the mean velocity at which the frequency perturbation travels. Therefore, V is not a vector-valued variable in this method. The unknowns in the above equation set are: the hypocentral coordinates, (x_e, y_e) , and the time at which the event occurred, t_e .

To find a set of solutions, we seek a linear system in terms of the hypocentral coordinates such that the least-squares method of solving an over-constrained system of equations can be used. In general, our system of equations will be over-constrained since many more FDRs typically respond than there are variables. By subtracting successive pairs of equations, a linear equation in terms of the hypocentral coordinates is produced:

$$(x_{i+1} - x_i)x_e - (y_{i+1} - y_i)y_e - V^2(t_{i+1} - t_i)t_e = C_i \quad (14)$$

where C_i is defined as:

$$C_i = \frac{1}{2} [V^2(t_{i+1} - t_i) + x_{i+1}^2 + y_{i+1}^2 - x_i^2 - y_i^2] \quad (15)$$

Hence, (13) can be written for every unit that responds forming a system of equations that can be placed in a matrix to form:

$$\mathbf{C} = \mathbf{H}\mathbf{x} \quad (16)$$

with matrix variables defined as:

$$\mathbf{C} = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_h \\ y_h \\ t_h \end{bmatrix} \quad (17)$$

$$\mathbf{H} = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & V^2(t_2 - t_1) \\ x_3 - x_2 & y_3 - y_2 & V^2(t_3 - t_2) \\ \vdots & \vdots & \vdots \\ x_n - x_{n-1} & y_n - y_{n-1} & V^2(t_n - t_{n-1}) \\ x_1 - x_n & y_1 - y_n & V^2(t_1 - t_n) \end{bmatrix} \quad (18)$$

To solve for \mathbf{x} in (16), which is constructed to contain the hypocentral coordinates, we use the pseudo-inverse defined as:

$$\mathbf{H}^\dagger = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \quad (19)$$

The final least squares solution is:

$$\mathbf{x} = \mathbf{H}^\dagger \mathbf{C} \quad (20)$$

The least squares based triangulation method is currently used in FNET and has worked well on over 1000 event location estimations performed since January 2006.

V. CASES REVIEW

A few examples of FNET event location estimation are provided below. In the following figures, the blue dot is the actual event location, and the red circle is the estimated event location scope found using the least squares method. The radius of the circle is about 100 miles.

A. Case 1

Case 1 happened at Watts Bar power plant on May 30, 2006, see Fig. 9a and 9b. The triangulation error is 79 miles.

B. Case 2

Case 1 happened at Limerick power plant on February 1, 2008, see Fig. 10a and 10b. The triangulation error is 40 miles.

C. Case 3

Case 3 happened at W H Zimmer power plant on February 19, 2009, see Fig. 11a and 11b. The triangulation error is 0, because we pinpointed the power plant in this case.

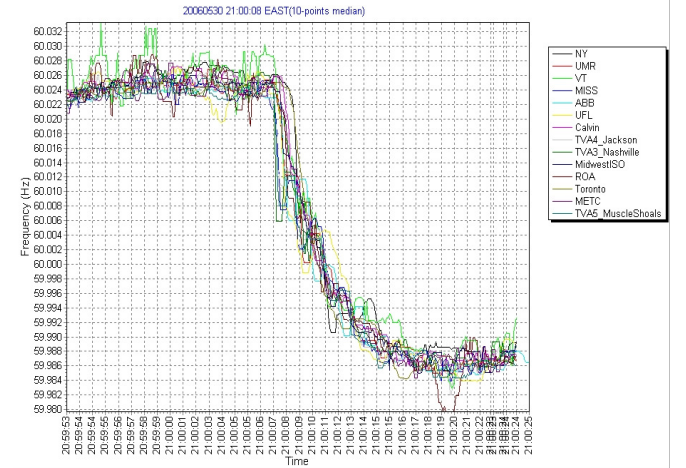


Fig. 9a. Event Frequency Plot of Case 1

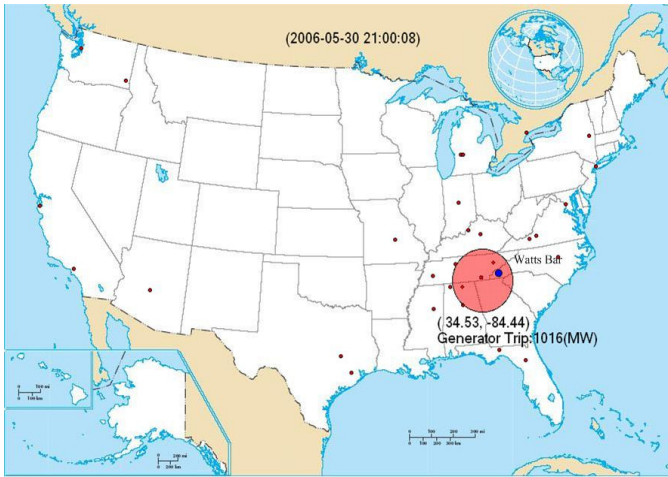


Fig. 9b. Triangulation results of case 1

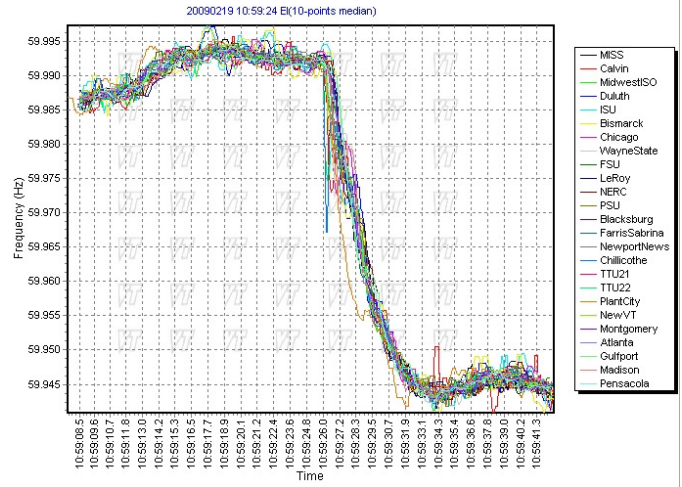


Fig. 11a. Event Frequency Pot of Case 3

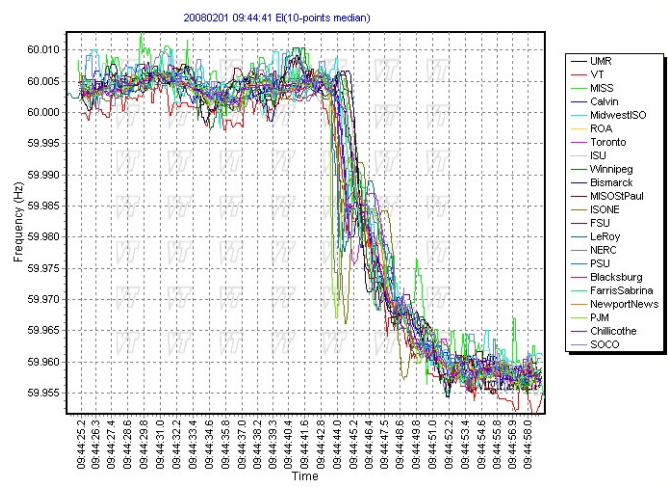


Fig. 10a. Event Frequency Plot of Case 2

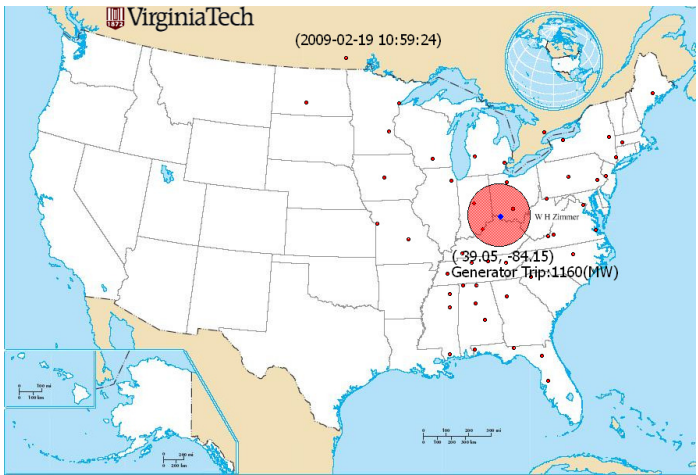


Fig. 11b. Triangulation results of case 3

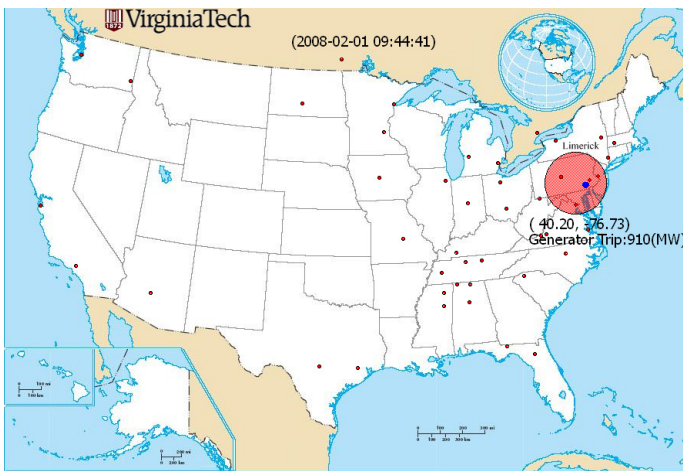


Fig. 10b. Triangulation results of case 2

VI. CONCLUSION

Taking advantage of the fast development of microprocessor technology and the commercialization of the GPS system, the Frequency Monitoring Network (FNET) group of Virginia Tech has established an extremely low cost and quickly deployable wide-area frequency measurement system with high dynamic accuracy to monitor the frequency behavior of the North America power grids using widely deployed frequency disturbance recorders (FDRs). During the 5 years since the first device was installed in 2003, the system has detected nearly one thousand frequency events and the analysis results have been sent to power utilities and research institutes, thus providing power system researchers, operators, customers, and policy makers an Internet accessible, cost-effective, cross-platform frequency information monitoring network.

Frequency perturbations after events like generation trips travel through a power system grid with finite speed and therefore arrive at particular Frequency Disturbance Recorders (FDRs) at different times. The FDRs sample the voltages at different locations; calculate the frequency, angle, and magnitudes of the voltage; and send the data to an Information Management System (IMS) through the Internet. Event

location algorithms can use data from this system to triangulate the location of an initiating event and estimate its size.

The event location engine uses a least squares method to detect events in real time from on-line data. The tool also incorporates on-line data processing, data plotting, and event display methods, as well as an event size estimation module developed in an earlier project.

This work has made power grid situation awareness monitoring based on distribution level frequency measurements a reality. This technology could also be an ideal tool for grid reliability coordinators and homeland security monitoring. Although this demonstration of feasibility uses data from FNET, the algorithms developed under this project can be implemented in any situation as long as accurate synchronous dynamic frequency data is available.

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